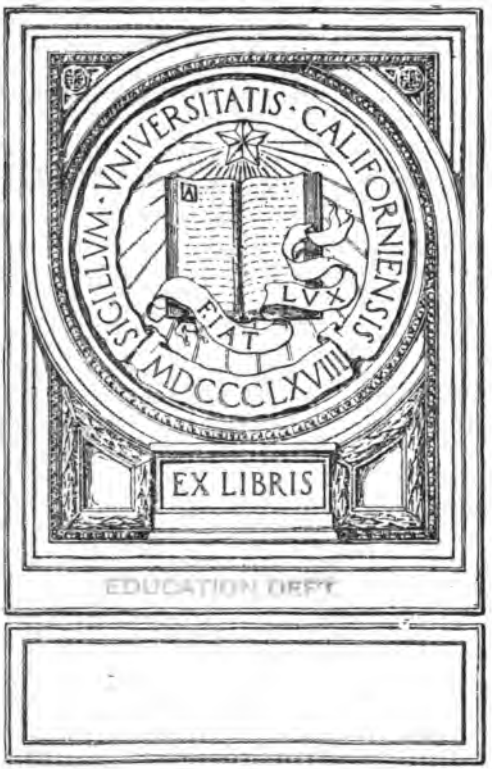
Browne's Graded School Arithmetic

BOOK TWO

THE WHITAKER & RAY COMPANY
PUBLISHERS
San Francisco


SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
LVX
FIAT
MDCCCLXVIII
EX LIBRIS
EDUCATION DEPT

GRADED SCHOOL ARITHMETIC

BOOK TWO

AN ELEMENTARY TEXT FOR USE IN PUBLIC AND PRIVATE SCHOOLS, FROM THE FIFTH TO THE EIGHTH YEAR, INCLUSIVE

BY

FRANK J. BROWNE

EX-SUPERINTENDENT PUBLIC INSTRUCTION OF THE STATE OF WASHINGTON; HEAD MASTER HOITT'S PREPARATORY SCHOOL, MENLO PARK, CALIFORNIA; AUTHOR OF BROWNE'S GRADED MENTAL ARITHMETIC

SAN FRANCISCO
THE WHITAKER AND RAY COMPANY
(INCORPORATED)
1903

PREFACE.

Text-books on arithmetic have been criticised by teachers and school supervisors for several reasons, among which are the following—

1. They contain unnecessary matter, by giving undue emphasis to impractical topics.

2. The gradation and pedagogical presentation of the subject is often made secondary to the logical arrangement of the material.

3. The many subdivisions of arithmetic, arising from the multitude of applications, often lead to a confused conception of the subject as a whole. Emphasis should be given rather to the few principles of relation, than to an extensive outline of subjects.

4. There is often too much discussion, and too little work.

5. Arithmetic is often considered as something apart from algebra and geometry, and no effort is made in the grammar grades to show its proper relation to those subjects.

6. The subject has too little relation to business life.

The GRADED SCHOOL ARITHMETIC, BOOK TWO, has been prepared to meet the above-named criticisms.

1. But little emphasis has been given to compound and annual interest, partial payments, foreign exchange, solid mensuration, and other topics, and thus much unnecessary matter has been eliminated.

2. The matter has been presented with a view to the gradual development of the pupil's mental power.

3. The several topics are related by the emphasis given to common principles. For example, Ratio and Simple Proportion are introduced before Percentage and Interest, because the principles of Ratio and Proportion are fundamental to the operations of Percentage and Interest.

4. Algebra and Geometry are presented in their relation to arithmetic, and in a large measure become a part of it.

5. The space given to discussions and rules has been minimized, and an abundance of problems has been provided.

6. The elementary method of keeping accounts will bring the pupil in touch with business life, and will train him in business habits.

But little need be said as to methods, aside from the plans indicated by the text itself. Whatever of "general method" is applicable to the teaching of arithmetic is stated in the following extracts from a bulletin on arithmetic, written by Prof. W. H. Baker of the State Normal School, San Jose, California:

"A pupil may be able to perform all the arithmetical operations, yet be helpless in an attempt to deal with a given problem. The *how* deals with the mechanics of number, and may be mastered by rule. The *what* has to do with mind, and can be mastered only as the mind grows. Any rule for this work would defeat the purpose of arithmetical study. Arithmetic does not need to wait on this growth. By well arranged exercises and proper methods it may and should minister to it."

"The wise teacher seeks to 'liberate' her pupils—to make them self-reliant, and to lead them to a point where they can proceed without further assistance. When a pupil has grasped the law on which a process depends, he is ready to become his own master so far as that process is concerned. It is the teacher's duty to lead the child to discover the law, and to fix that law by appropriate exercises."

As to typography, much care has been exercised to make this book superior to others in artistic finish. Its attractive style adds to its value as a text.

With the hope that it will inspire deeper interest in the subject, and will be of service to all who may use it, the consideration of schools and teachers everywhere is invited.

FRANK J. BROWNE.

SAN FRANCISCO, CALIFORNIA,
June 1, 1903.

COURSE OF STUDY.

This book follows BOOK ONE in the common school course, and completes the arithmetic of the grammar grades. It may be introduced in the latter part of the fifth year, upon completing BOOK ONE, and graded as follows:

ORIGINAL PROBLEMS AND REVIEW EXERCISES THROUGHOUT THE COURSE.

DEFINITIONS.

QUANTITY is anything that has one or more dimensions. It can be increased, diminished or measured.

A UNIT is a single thing, or a standard by which quantity is measured.

A NUMBER shows how many units are in a given quantity.

AN ABSTRACT UNIT is simply *one*, without reference to any object or quantity. An abstract number is composed of abstract units.

A CONCRETE or DENOMINATE unit is a particular object or quantity used as a standard to measure another quantity of the same kind. A denominate number is composed of denominate units. A COMPOUND denominate number is expressed by two or more denominations; as, 2 pounds 8 ounces.

AN INTEGER is an entire unit.

A FRACTION is a part of a unit. If a unit is divided into four parts, one of the parts is called one *fourth;* if it be divided into five parts, one of the parts is called one *fifth*, etc. A fraction is expressed by two figures, one above and the other below a short horizontal line. Thus, one fifth is written $\frac{1}{5}$; three fifths is written $\frac{3}{5}$, etc.

The number below the line tells into how many parts the unit is divided, and is called the *denominator.* The number above the line tells how many of these parts are considered, and is called the *numerator.*

ARITHMETIC treats of numbers and how to use them in computation.

THE ARABIC method of expressing numbers is the one in common use. It employs the following ten characters:

One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Naught
1	2	3	4	5	6	7	8	9	0

The first nine figures are called *digits*, or significant figures.

NOTATION AND NUMERATION.

NOTATION is the art of writing numbers.

NUMERATION is the art of reading numbers.

A significant figure always represents the same number of units, but the units are not always of the same value. A figure standing alone has its simple value. When used with other figures it has its local value, determined by the place in which it is used.

Thus, 2,345 represents a number, and each figure expresses units, but units of different value. The 5 is in units' place, or first order, and has the simple value of five *ones*. The 4 has the second place from the right, or the second order, a unit of which has ten times the value of a unit of the first order, and it represents four *tens*. The 3 has the third place from the right, or third order, a unit of which has ten times the value of a unit of the second order, and one hundred times the value of a unit of the first order, and it represents 3 *hundreds*. The 2 has the fourth place, or fourth order, a unit of which has ten times the value of a unit of the third order, one hundred times the value of a unit of the second order, and one thousand times the value of a unit of the first order, and it represents 2 *thousands*.

The NAUGHT, also called CIPHER or ZERO, signifies the absence of number. The figure 5, if written alone, represents 5 *units*. If it is written with a naught at the right, 50, it represents 5 *tens*, and the 0 signifies the absence of units. When any order has no significant figure, a *naught* must be written to fill the order. In the number 109, the 0 must be placed in tens' order to keep the 1 and the 7 in their proper places.

For convenience in reading numbers the orders are separated into groups of three places each, as indicated by the

NUMERATION TABLE.

hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units	hundreds	tens	units
of sextillions.			of quintillions.			of quadrillions.			of trillions.			of billions.			of millions.			of thousands.				

It is seen that the figure occupying any place represents ten times the value it would represent if it stood one place farther to the right, and one tenth the value it would represent if it stood one place farther to the left; one hundred times the value it would represent if it stood two places to the right, and one hundredth the value it would represent it it stood two places to the left, etc. This relation is called the *decimal system* of notation, and has the uniform *scale of ten.* This principle extended would make the first figure at the right of units represent *tenths;* the second, *hundredths;* the third, *thousandths*, etc., as follows:

Thousands.	Hundreds.	Tens.	Units.	Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.	Ten-millionths.	Hundred-millionths.	Billionths.
0	0	0	0	.	0	0	0	0	0	0	0	0	0

Numbers thus expressed by figures at the right of the point are called *decimal fractions*, because their denominator is always

10, 100, 1000, or some other power of 10, the word *decimal* being derived from the Latin word *decem*, meaning ten. If the denominator is written, the decimal takes the form of a *common fraction.*

HOW TO READ AND WRITE NUMBERS.

To read a number expressed by figures, divide it into periods of three places each, beginning at the right; then, beginning at the left, read the figures of each period, and give them the name of the period.

EXAMPLE—23, 043, 512, 039, 578 is read 23 *trillions*, 43 *billions*, 512 *millions*, 39 *thousands*, 578.

Numbers containing both *integers* and *decimal fractions* are called *mixed decimals.* They are read by first reading the figures at the left of the point as if they stood alone, and then reading the figures at the right of the point, giving them the name of the order in which the right-hand figure is written.

EXAMPLE—25.25 is read 25 and 25 *hundredths;* 3.016 is read 3 and 16 *thousandths.* Always read "and" between the integer and the decimal.

To write numbers by figures, write in each successive period the figures which represent the number of the orders of that period, writing a cipher in each vacant place. The left-hand period may have one, two, or three figures; each of the other periods must have three.

To write decimal fractions, or decimals, by figures, write them as if they were integers, then place the decimal point so as to bring the right-hand figure in the order having the name of the denominater of the fraction.

EXAMPLE—3 and 13 thousandths is written 3.013, bringing the right-hand 3 in thousandths' order.

There are always as many places at the right of the point as there are ciphers used in writing the denominator of the fraction. When there are not figures enough for this in the given decimal, ciphers must be prefixed.

Read the following numbers, and afterward write them from dictation—

1.	678,000	**10.**	20,020,020,020
2.	5,664,100	**11.**	78,780,780,078
3.	50,500,500	**12.**	234,345,456,678
4.	15,015,015	**13.**	1,004,005,007,009
5.	70,007,007	**14.**	18,081,071,061,116
6.	700,070,070	**15.**	50,050,050,050,050
7.	77,077,077	**16.**	505,505,505,505,505
8.	808,800,008	**17.**	150,015,015,015,015
9.	90,099,999	**18.**	23,003,300,030,003

19.	.1	**35.**	990.0111
20.	.01	**36.**	999.1111
21.	1.1	**37.**	678.6789
22.	10.1	**38.**	123.4567
23.	11.01	**39.**	345.0789
24.	50.5	**40.**	304.0506
25.	55.05	**41.**	7.0007
26.	505.005	**42.**	71.0017
27.	678.55	**43.**	711.0117
28.	123.555	**44.**	9,876.6118
29.	600.009	**45.**	1,011.00111
30.	1.111	**46.**	555.12121
31.	11.011	**47.**	6,000.00008
32.	101.001	**48.**	707.080808
33.	100.0001	**49.**	1,040.0040044
34.	90.0011	**50.**	800.8088888

Read at sight—

51.	The planet Mercury	is	36,814,720 miles from the sun.
52.	The planet Venus	is	68,791,750 miles from the sun.
53.	The Earth	is	91,600,000 miles from the sun.
54.	The planet Mars	is	144,907,630 miles from the sun.
55.	The planet Jupiter	is	494,499,100 miles from the sun.

MULTIPLYING AND DIVIDING BY 10, 100, 1000, ETC.

The principles of the decimal system of writing numbers show that by removing the decimal point of any number one place to the right, the number is multiplied by 10; by removing it one place to the left, the number is divided by 10. Removing the point two places to the right, multiplies by 100; two places to the left, divides by 100, etc. Thus, in the number 65.34, by removing the point

One place to the right gives 653.4, or 10 times 65.34;
One place to the left gives 6.534, or $\frac{1}{10}$ of 65.34;
Two places to the right gives 6534, or 100 times 65.34;
Two places to the left gives .6534, or $\frac{1}{100}$ of 65.34.

Hence, to divide by 10, 100, 1000, etc., remove the decimal point as many places to the left as there are ciphers in the divisor; to multiply by 10, 100, 1000, etc., remove the decimal point as many places to the right as there are ciphers in the multiplier.

EXAMPLE—$49.1 \times 10 = 491.$ $\quad 49.1 \div 10 = 4.91$
$49.1 \times 100 = 4910.$ $\quad 49.1 \div 100 = .491$
$49.1 \times 1000 = 49100.$ $\quad 49.1 \div 1000 = .0491$

State at sight the products—

1.	78	$\times 10$	**13.**	.02	$\div 10$
2.	78	$\div 10$	**14.**	.25	$\div 100$
3.	78.3	$\times 100$	**15.**	1.125	$\div 100$
4.	3.87	$\div 100$	**16.**	.375	$\div 1000$
5.	268	$\div 100$	**17.**	.008	$\times 100$
6.	26.8	$\times 1000$	**18.**	.0006	$\times 1000$
7.	263.7	$\div 1000$	**19.**	.0101	$\times 100$
8.	35	$\div 1000$	**20.**	.2002	$\times 10000$
9.	7	$\div 1000$	**21.**	20.02	$\div 10000$
10.	8.5	$\times 1000$	**22.**	1.987	$\div 1000$
11.	85.5	$\div 1000$	**23.**	19.87	$\div 10000$
12.	.085	$\times 1000$	**24.**	.198	$\div 1000$

ROMAN NOTATION.

Roman Notation uses seven capital letters to express numbers, viz. :—

Letters	I	V	X	L	C	D	M
Values	1	5	10	50	100	500	1000

These letters are used according to the following principles—

1. Repeating a letter repeats its value. Thus XX represents 20; CC, 200; III, 3.

2. A letter placed after one of greater value is added to the greater. Thus VI represents 6; LX, 60; DC, 600.

3. A letter placed before one of greater value is taken from the greater. Thus IV represents 4; XL, 40; CD, 400.

4. A letter placed between two letters of greater value is taken from the sum of the other two. Thus XIV represents 14; XIX, 19.

5. A bar placed over a letter increases its value 1000 times. Thus $\overline{V}$ represents 5000; $\overline{L}$, 50,000.

TABLE OF ROMAN NOTATION.

I = 1	XI = 11	XXI = 21	CC =	200
II = 2	XII = 12	XXII = 22	D =	500
III = 3	XIII = 13	XXX = 30	DC =	600
IV = 4	XIV = 14	XL = 40	M =	1000
V = 5	XV = 15	L = 50	MC =	1100
VI = 6	XVI = 16	LX = 60	MD =	1500
VII = 7	XVII = 17	LXX = 70	MM =	2000
VIII = 8	XVIII = 18	LXXX = 80	$\overline{X}$ =	10000
IX = 9	XIX = 19	XC = 90	$\overline{C}$ =	100000
X = 10	XX = 20	C = 100	$\overline{M}$ =	1000000

Roman Notation is used principally in numbering chapters, lessons, and sections in books and printed documents.

DENOMINATE NUMBERS.

In denominate numbers, the relation between the several orders or units is not based upon the decimal system, as in abstract numbers, but varies at each step of the scale. Thus, in the compound number

2 bu. 2 pk. 2 qt. 1 pt.,

2 pints make 1 quart, a unit of the next higher order;
8 quarts make 1 peck, a unit of the next higher order;
4 pecks make 1 bushel, a unit of the next higher order;

Instead of a uniform scale of 10, as in abstract numbers, the quantity has a *varying* scale. Instead of 10 units of one order making one of the next higher, the scale is seen to be 2, 8, and 4. Other measures have still other scales by which their several units are expressed.

In addition to the abstract unit, *one*, arithmetic treats of several other kinds of units used in measuring denominate quantities. They are

Units of value, or money.
Units of capacity.
Units of length.
Units of surface.
Units of volume.
Units of weight.
Units of time.
Units of angular measure.

No school should be without the proper objects by which the absolute and relative values of the various units may be learned by sight and use. These objects should include

Toy money;
Gill, pint, quart, and gallon measures;
Peck and bushel measures;
Foot rules and yard sticks;
Cubical blocks;
Balances, with ounce and pound weights;
Clock;
Protractor for measuring angles.

UNITS OF VALUE.

UNITED STATES CURRENCY.

10 mills make 1 cent (¢ *or ct.*)
10 cents make 1 dime.
10 dimes make 1 dollar ($)
10 dollars make 1 eagle (*E.*)

			¢	mills
		dimes	1 =	10
	$	1 =	10 =	100
E.	1 =	10 =	100 =	1000
1 =	10 =	100 =	1000 =	10000

This table of units has the uniform scale of ten, and any sum of money should be treated as a simple quantity. This is also true of

Japanese currency:	100 sen	= 1 yen	= $0.81
Austrian currency:	100 kreutzers	= 1 florin	= $0.453
Russian currency:	100 kopecks	= 1 rouble	= $0.734
German currency:	100 pfenning	= 1 mark	= $0.238
Italian currency:	100 centissimi	= 1 lira	= $0.193
French currency:	100 centimes	= 1 franc	= $0.193

ENGLISH CURRENCY.

4 farthings (*far.*) make 1 penny (*d.*)
12 pence make 1 shilling (*s.*)
20 shillings make 1 pound (£)
21 shillings make 1 guinea.
5 shillings make 1 crown.

A pound (£) or sovereign is equivalent to $4.8665.

NOTE.—The symbol £ is used because it is the first letter of the Latin word *libra*, which signifies a pound; *s*, stands for *solidus*, which signifies a shilling; *d*, for *denarius*, a penny.

UNITS OF CAPACITY.

DRY MEASURE.

Dry measure is used in measuring articles in bulk, as grain, vegetables, fruit, etc.

TABLE.

2 pints (*pt.*) make 1 quart (*qt.*)
8 quarts make 1 peck (*pk.*)
4 pecks make 1 bushel (*bu.*)

		qt.	pt.
	pk.	1 =	2
bu.	1 =	8 =	16
1 =	4 =	32 =	64

A dry quart contains 67.2 cu. in.
A bushel contains 2150.4 cu. in.

LIQUID MEASURE.

Liquid measure is used in measuring liquids.

TABLE.

4 gills (*gi.*) make 1 pint (*pt.*)
2 pints make 1 quart (*qt.*)
4 quarts make 1 gallon (*gal.*)
31½ gallons make 1 barrel (*bbl.*)
2 barrels make 1 hogshead (*hhd.*)

			pt.	gi.
		qt.	1 =	4
	gal.	1 =	2 =	8
bbl.	1 =	4 =	8 =	32
1 =	31½ =	126 =	252 =	1008

The hogshead of 63 gallons is used in estimating the contents of reservoirs, or other large quantities. In most other cases the hogshead is an indefinite quantity. Large casks containing 50, 60, 100, or more gallons, are called hogsheads.

A liquid quart	contains	57.75	cu. in.
A gallon	contains	231	cu. in.

APOTHECARIES' FLUID MEASURE.

60 minims or drops (*m.*)	= 1 fluid dram (f ʒ)
8 fluid drams	= 1 fluid ounce (f ℥)
16 fluid ounces	= 1 pint (O.)
8 pints	= 1 gallon (Cong.)

UNITS OF LENGTH.

There are three kinds of extension—length, breadth, and thickness. A *line* has but the one dimension of *length*. A *surface* has two dimensions—*length* and *breadth*. A *solid* has three dimensions—*length*, *breadth*, and *thickness*.

LONG OR LINEAR MEASURE is used in measuring length and distance.

TABLE.

12	inches (*in.*)	make 1 foot (*ft.*)
3	feet	make 1 yard (*yd.*)
5½	yards	make 1 rod (*rd.*)
40	rods	make 1 furlong (*fur.*)
8	furlongs, or 320 rods	make 1 mile (*mi.*)

				ft.	in.
			yd.	1 =	12
		rd.	1 =	3 =	36
	fur.	1 =	5½ =	16½ =	198
mi.	1 =	40 =	220 =	660 =	7920
1 =	8 =	320 =	1760 =	5280 =	63360

For land surveying the units are arranged on another scale—

7.92 inches (*in.*) make 1 link (*l.*)
100 links (4 rds.) make 1 chain (*ch.*)
80 chains make 1 mile (*mi.*)

Mariners use the following units—

6 feet make a fathom.
120 fathoms make 1 cable's length.
1 nautical mile, or knot, makes 1.15 common miles.
3 nautical miles make 1 league.

69.16 common miles make 1 degree of latitude, or of longitude at the equator.

A KNOT is used in measuring the speed of vessels.

A HAND is 4 inches, used in measuring the height of horses.

A SIZE is $\frac{1}{8}$ of an inch, used in measuring boots and shoes.

A FURLONG is an old road measure, now but little used.

UNITS OF SURFACE.

SQUARE MEASURE is used in measuring surfaces, or areas.

TABLE.

144 square inches (*sq. in.*) make 1 square foot (*sq. ft.*)
9 square feet make 1 square yard (*sq. yd.*)
30¼ square yards make 1 square rod (*sq. rd.*)
160 square rods make 1 acre (*A.*)
640 acres (one section) make 1 square mile (*sq. mi.*)

			sq. ft.		sq. in.
		sq. yd.	1	=	144
	sq. rd.	1	= 9	=	1296
acre	1	= 30¼	= 272¼	=	39204
1	= 160	= 4840	= 43560	=	6272640

For land surveying the following units are used—

16 square rods make 1 square chain.
10 square chains make 1 acre.

NOTE.—Since a chain is $\frac{1}{10}$ of an acre, chains may be written as a decimal part of an acre. Thus, 2 A. 4 chains may be written 2.4 acres.

UNITS OF VOLUME.

SOLID MEASURE is used in measuring solids or spaces, having length, breadth, and thickness.

TABLE.

1728	cubic inches (*cu. in.*)	make 1 cubic foot (*cu. ft.*)
27	cubic feet	make 1 cubic yard (*cu. yd.*)
128	cubic feet	make 1 cord of wood (*cd.*)
24¾	cubic feet	make 1 perch of stone.

NOTE.—A pile of wood 4 ft. wide, 4 ft. high, and 8 ft. long makes a cord. One foot in length of such a pile is called a cord foot. It contains 16 solid feet.

UNITS OF WEIGHT.

AVOIRDUPOIS WEIGHT.

This is the common weight used in weighing articles in bulk, such as groceries, fruit, grain, etc.; also all metals except gold, silver, and jewels.

TABLE.

16 drams (*dr.*)	make 1 ounce (*oz.*)
16 ounces	make 1 pound (*lb.*)
100 pounds	make 1 hundredweight (*cwt.*)
20 cwt. (2000 lb.)	make 1 ton (*T.*)

The *dram* is not used in business transactions.

		lb.	oz.	dr.
		1 =	1 =	16
	cwt.	1 =	16 =	256
T.	1 =	100 =	1600 =	25600
1 =	20 =	2000 =	32000 =	512000

100 pounds is also called a *cental*, used in weighing grain.

The standard avoirdupois pound is the weight of 27.7015 cubic inches of distilled water. A cubic foot of water weighs *very nearly* 1,000 ounces.

By the old method of weighing, taken from the English system, 112 pounds were reckoned a hundredweight, and 2,240 pounds a

ton; but this is seldom used except in weighing English imports at the custom-house, and in freighting coal and iron at the mines.

These are called the *long ton* and the *long hundredweight.*

MISCELLANEOUS WEIGHTS.

A bushel of corn or rye	= 56 pounds.
A bushel of corn or rye meal	= 50 pounds.
A bushel of wheat or beans	= 60 pounds.
A bushel of potatoes	= 60 pounds.
A bushel of oats	= 32 pounds.
A bushel of barley	= 48 pounds.
A bushel of timothy seed	= 45 pounds.
A barrel of flour	= 196 pounds.
A barrel of pork or beef	= 200 pounds.
A cask of lime	= 240 pounds.
A keg of nails	= 100 pounds.
A quintal of fish	= 100 pounds.

TROY WEIGHT.

Gold, silver, and jewels are weighed by troy weight.

TABLE.

24 grains (*gr.*) make 1 pennyweight (*pwt.*)
20 pennyweights make 1 ounce (*oz.*)
12 ounces make 1 pound (*lb.*)

		pwt.	gr.
	oz.	1 =	24
lb.	1 =	20 =	480
1 =	12 =	240 =	5760

Note.—Practical illustrations of troy weight are to be found in the United States coins. The gold dollar weighs 25.8 grains; the silver dollar, 412½ grains.

Gold and silver are bought and sold by the ounce. The *carat*, very nearly equal to 3⅛ troy grains, is used in weighing diamonds and other precious stones.

The word *carat* is also used in expressing the number of parts of pure gold in articles of jewelry, etc. If 18 parts out of 24 are pure gold, and the remaining 6 parts are alloy, the metal is said to be 18 carats fine; 15 parts out of 24 is 15 carats, etc.

The troy pound deposited in the mint at Philadelphia is the standard of weight.

It is the weight of 22.794377 cubic inches of distilled water. Hence it is less than the pound avoirdupois.

7000 troy grains = 1 avoirdupois pound.
175 troy pounds=144 avoirdupois pounds.
175 troy ounces =192 avoirdupois ounces.
437½ troy grains = 1 avoirdupois ounce.

Note that the avoirdupois pound is the larger pound, and that the troy ounce is the larger ounce.

APOTHECARIES' WEIGHT.

This weight is used by druggists and physicians in mixing their medicines. When medicines are sold in bulk, avoirdupois weight is used.

TABLE.

20 grains (*gr.*) make 1 scruple (℈).
3 scruples make 1 dram (ʒ).
8 drams make 1 ounce (℥).
12 ounces make 1 pound (℔).

		℈	gr.
	ʒ	1 =	20
℥	1 =	3 =	60
lb. 1 =	8 =	24 =	480
1 = 12 =	96 =	288 =	5760

The pound, ounce, and grain have the same weight as the pound, ounce, and grain in troy weight.

Note—The *grain* originally was the weight of a grain of well-dried wheat from the middle of the ear. At first 32, and afterward 24, of these grains were used to determine the weight of an English penny. It thus became the unit of the English system of weights, from which are derived the several units of troy, avoirdupois, and apothecaries' weight.

UNITS OF TIME.

TIME is a measured part of duration.

TABLE.

60 seconds (*sec.*)	make 1 minute (*min.*)
60 minutes	make 1 hour (*hr.*)
24 hours	make 1 day (*da.*)
7 days	make 1 week (*wk.*)
365 days	make 1 common year (*yr.*)
366 days	make 1 leap year.
12 calendar months	make 1 year.

				min.	sec.
			hr.	1 =	60
		da.	1 =	60 =	3600
	wk.	1	24 =	1440 =	86400
	1 =	7 =	168 =	10080 =	604800
1 yr. = 12 mo. =		365 =	8760 =	525600 =	31536000
		366 =	8784 =	527040 =	31622400

The months and seasons are as follows—

Winter	January.	Jan.	31 days.
	February.	Feb.	28 or 29 days.
Spring	March.	Mar.	31 days.
	April.	Apr.	30 days.
	May.	May.	31 days.
Summer	June.	June.	30 days.
	July.	July.	31 days.
	August.	Aug.	31 days.
Autumn	September.	Sept.	30 days.
	October.	Oct.	31 days.
	November.	Nov.	30 days.
Winter	December.	Dec.	31 days.

NOTE—**In business transactions 30 days usually make a month, and 360 days make a year. See Interest and Bank Discount.**

Memorize—

Thirty days hath September,
April, June, and November;
All the rest have thirty-one
Save February, which alone
Hath twenty-eight, and one day more
We add to it each year in four.

NOTE 2—The exact length of a solar year is 365 days 5 hours 48 minutes 46 seconds; but for convenience it is reckoned 11 min. 14 sec. more than this, or 365 da. 6 hr. = 365¼ days. This ¼ day in 4 years makes one day, which, every fourth or leap year, is added to February, giving it 29 days.

The following rule for leap year will render the calendar correct for a period of several thousand years.

1. Every year exactly divisible by 4 is a leap year, the centennial years excepted.

Thus, 1876 is a leap year, but 1877 is a common year.

2. Every centennial year exactly divisible by 400 is a leap year.

Thus the year 2000 is a leap year, but 1800 and 1900 are common years.

UNITS OF ANGULAR MEASUREMENT.

Every circumference is divided into 360 equal parts, called degrees. The actual length of a degree depends upon the size of the circumference.

TABLE.

60 seconds (″) make 1 minute(′)
60 minutes make 1 degree (°)
30 degrees make 1 sign (*s.*)
12 signs make 1 circle (*c.*)

		°	′	″
			1 =	60
	s.	1 =	60 =	3600
c.	1 =	30 =	1800 =	108000
1 =	12 =	360 =	21600 =	1296000

NOTE—The *sign* is used in astronomy. One *minute* of the circumference of the earth constitutes 1 geographical mile, which is about 1.15 common miles.

The sun appears to move westward over an entire circumference of 360° in 24 hours. Hence, it appears to move over $\frac{1}{24}$ of 360° or 15° in 1 hour. If the difference of longitude between two places is 15°, the difference of time between the two places is 1 hour. If it be noon at the more western place, it will be 1:00 P. M. at the other place. If it be noon at the more eastern place, it will be 11:00 A. M. at the other place. In all cases, the time of two places will differ at the rate of 1 hour for every 15° of longitude between the two places.

MISCELLANEOUS TABLES.

FOR COUNTING.

12 units make 1 dozen (*doz.*)
12 dozen make 1 gross.
12 gross make 1 great gross.
20 units make 1 score.

FOR PAPER.

24 sheets make 1 quire.
20 quires make 1 ream.
2 reams make 1 bundle.
5 bundles make 1 bale.

		quires	sheets
	reams 1 =		24
	bundles 1 =	20 =	480
bale 1 =	2 =	40 =	960
1 = 5 =	10 =	200 =	4800

FOR BOOKS.

A book formed of sheets folded—

In 2 leaves is called a folio	= 4 pp. (*pages*)
In 4 leaves is called a quarto, (*4to*)	= 8 pp.
In 8 leaves is called an octavo, (*8vo*)	= 16 pp.
In 12 leaves is called a duodecimo, (*12mo*)	= 24 pp.
In 16 leaves is called a 16mo	= 32 pp.
In 18 leaves is called an 18mo	= 36 pp.
In 24 leaves is called a 24mo	= 48 pp.
In 32 leaves is called a 32mo	= 64 pp.

BOOK TWO.

GRADED SCHOOL ARITHMETIC.

INTRODUCTORY EXERCISES.

1. Count by 2's from 2 to 24, and back from 24 to 2.
2. Count by 2's from 1 to 23, and back from 23 to 1.
3. Count by 3's from 3 to 36, and back from 36 to 3.
4. Count by 3's from 2 to 35, and back from 35 to 2.
5. Count by 3's from 1 to 34, and back from 34 to 1.
6. Count by 4's from 4 to 48, and back from 48 to 4.
7. Count by 4's from 3 to 47, and back from 47 to 3.
8. Count by 4's from 2 to 46, and back from 46 to 2.
9. Count by 4's from 1 to 45, and back from 45 to 1.
10. A boy had 6 cents, found 3, and his mother gave him 4 cents more. How many had he then?
11. Bryant had 6 marbles, bought 5, and found 3 more. How many had he then?
12. A girl had 25 cents. After spending 5 cents for a pencil and 12 cents for a book, how many cents had she left?
13. A boy paid 50 cents for a book, and 25 cents for some paper. What did he pay for both?
14. One book costs 50 cents and another costs 20 cents. The first cost how much more than the second?
15. A boy paid 5 cents car fare, 15 cents for lunch, and 10 cents for candy. The candy cost how much less than the lunch and car fare?
16. Four boys bought 6 oranges apiece, and lost one third of them. How many oranges were left?

1. Count by 5's from 5 to 60, and back from 60 to 5.
2. Count by 5's from 4 to 59, and back from 59 to 4.
3. Count by 5's from 3 to 58, and back from 58 to 3.
4. Count by 5's from 2 to 57, and back from 57 to 2.
5. Count by 5's from 1 to 56, and back from 56 to 1.
6. Count by 6's from 6 to 72, and back from 72 to 6.
7. Count by 6's from 5 to 71, and back from 71 to 5.
8. Count by 6's from 4 to 70, and back from 70 to 4.
9. Count by 6's from 3 to 69, and back from 69 to 3.
10. Count by 6's from 2 to 68, and back from 68 to 2.
11. Count by 6's from 1 to 67, and back from 67 to 1.
12. At 6 cents a quart, what will be the cost of a gallon of milk? Of one half of a gallon?
13. At the rate of 4 pencils for 5 cents, what is the cost of one dozen pencils?
14. How many feet are in 5 yards? In one third of a yard? In $2\frac{1}{3}$ yards?
15. A boy having 16 marbles lost 12. What part of a dozen had he remaining?
16. A man bought a hat which cost \$4.50, and gave a \$10 bill in payment. How much change should he receive?
17. If 8 yards are cut from a rope 16 yards long, how many feet remain?
18. Of $1\frac{1}{2}$ dozen eggs, 8 were broken. How many eggs were left?
19. Willie wrote 12 lines in his copy book, and Henry wrote 4 lines more than Willie. How many did both write?
20. Albert had 10 marbles, and Edwin had 8 more than Albert. How many had both?
21. In one class were 8 boys, and in another class were 9 boys. How many more than a dozen in both classes?
22. John is 16 years of age, and his brother is 6 years younger. What is the sum of their ages?
23. How many days in 5 weeks? In 9 weeks?
24. How many school days in 7 weeks? In 11 weeks?

1. Count by 7's from 7 to 84, and back from 84 to 7.

2. Count by 7's from 6 to 83, and back from 83 to 6.

3. Count by 7's from 5 to 82, and back from 82 to 5.

4. Count by 7's from 4 to 81, and back from 81 to 4.

5. Count by 7's from 3 to 80, and back from 80 to 3.

6. Count by 7's from 2 to 79, and back from 79 to 2.

7. Count by 7's from 1 to 78, and back from 78 to 1.

8. At the rate of 5 miles an hour, how far will a man ride on a bicycle in $2\frac{1}{2}$ hours?

9. How many pecks in 3 bushels? In $\frac{1}{2}$ bu.? In $2\frac{1}{2}$ bu.?

10. At 6 cents each, what is the cost of $1\frac{1}{2}$ dozen copy books?

11. At 6 cents a yard, find the cost of $6\frac{2}{3}$ yards of muslin.

12. At \$5 each, what is the cost of $2\frac{1}{2}$ dozen hats?

13. What part of a dollar are $2\frac{1}{2}$ dimes?

14. At \$7 a pair, find the cost of a dozen pairs of shoes.

15. At $1\frac{1}{2}$ cents for each egg, find the cost of 2 dozen eggs.

16. How many quarts in 2 pecks? In $\frac{1}{2}$ peck? In $2\frac{1}{2}$ pecks?

17. Ella paid 16 cents for 4 pencils. What would 1 dozen cost her at that rate?

18. How many bunches of pencils, with 5 in each bunch, can be made with 33 pencils? How many remain?

19. If I pay 5 cents for a pencil, how many pencils can I buy for one half-dollar? For 60 cents?

20. At 7 cents a yard, how many yards of muslin can be bought for 28 cents? For 70 cents? For \$1.40?

21. At 8 cents a quart, how many gallons of milk can be bought for 16 cents? For 32 cents? For 40 cents?

22. At \$3 a ton, how many tons of coal can be bought for \$12? For \$15? For \$16?

23. If 3 yards of velvet cost \$18, what is the price of 6 yards? Of 1 yard? Of 10 yards?

24. A man pays \$6 a month for his room. What rent will he pay for 6 months? For 15 days? For $10\frac{1}{2}$ months?

25. An office rents for \$10 a month. What is the rent for a year and a half?

1. Count by 8's from 8 to 96, and back from 96 to 8.
2. Count by 8's from 7 to 95, and back from 95 to 7.
3. Count by 8's from 6 to 94, and back from 94 to 6.
4. Count by 8's from 5 to 93, and back from 93 to 5.
5. Count by 8's from 4 to 92, and back from 92 to 4.
6. Count by 8's from 3 to 91, and back from 91 to 3.
7. Count by 8's from 2 to 90, and back from 90 to 2.
8. Count by 8's from 1 to 89, and back from 89 to 1.
9. If 4 pounds of meat cost 80 cents, what is the cost of 4 ounces at the same rate?
10. John received 50 cents for 5 hours' work. What would he be paid for 2 days' work of 10 hours each, at the same rate?
11. A teacher divided 42 marbles equally among 7 boys. What part of a dozen did each boy receive?
12. At the rate of $48 for 6 barrels of flour, what is the cost of one half of a barrel?
13. Find the value of 2 cords of wood, if 7 cords cost $28.
14. How many times must you cut a string 10 inches long to make five equal pieces 2 inches long?
15. James had a dime, spent a nickel for a pencil and 3 cents for pens. How many marbles could he then buy at the rate of 15 for 5 cents?
16. If two men can do a piece of work in four days, in what time can one man do it?
17. If a man can do a piece of work in eight days, in what time can four men do it?
18. The distance between two cities is 40 miles. At the rate of 5 miles an hour, in what time can a man go from one city to the other and half-way back?
19. Two $20 bills equal how many $5 bills?
20. If 8 pounds cost a dollar, what will 48 pounds cost?
21. If 16 pounds cost 48 cents, what will 8 pounds cost?
22. If 8 pounds cost 24 cents, what will 48 pounds cost?
23. If 16 pounds cost 48 cents, what will 24 pounds cost?
24. What 5 pieces of money make 75 cents? 80 cents?

1. Count by 9's from 9 to 108, and back from 108 to 9.
2. Count by 9's from 8 to 107, and back from 107 to 8.
3. Count by 9's from 7 to 106, and back from 106 to 7.
4. Count by 9's from 6 to 105, and back from 105 to 6.
5. Count by 9's from 5 to 104, and back from 104 to 5.
6. Count by 9's from 4 to 103, and back from 103 to 4.
7. Count by 9's from 3 to 102, and back from 102 to 3.
8. Count by 9's from 2 to 101, and back from 101 to 2.
9. Count by 9's from 1 to 100, and back from 100 to 1.
10. What 6 pieces of money make 50 cents?
11. Change a dollar so as to pay a bill of 72 cents.
12. How many pecks in $10\frac{1}{2}$ bushels?
13. How many quarts in 4 pecks 5 quarts?
14. Make change to pay 33 cents out of $5.
15. What part of 10 is 5? What part of 15 is 3?
16. If 15 pencils cost 30 cents, what will 3 pencils cost?
17. If 15 pencils cost 30 cents, what will 6 pencils cost?
18. If 15 oranges cost 30 cents, what will 9 oranges cost?
19. If 15 oranges cost 30 cents, what will 5 oranges cost?
20. If 15 oranges cost 30 cents, what will 10 oranges cost?
21. A grocer buys apples at 60 cents a bushel, and sells them at the rate of 20 cents a peck. What is his profit on 4 bushels?
22. Find the cost of four fifths of 30 pencils at 10 cents a dozen.
23. Albert has 25 cents, and Henry has 33 cents. How many cents must Henry give to Albert that they may have equal amounts?
24. If 6 oranges cost 18 cents, what will 18 oranges cost at the same rate?
25. If you have a quarter, a dime, a nickel, and 2 cents, how much have you?
26. Tell how to change a half-dollar to pay a bill of 18 cents.
27. In how many ways can you draw a rectangle having a surface of 24 square inches?
28. If 4 quarts of berries cost 20 cents, what will 5 quarts cost?

1. Count by 12's from 12 to 144, and back from 144 to 12.
2. Count by 12's from 11 to 143, and back from 143 to 11.
3. Count by 12's from 10 to 142, and back from 142 to 10.
4. Count by 12's from 9 to 141, and back from 141 to 9.
5. Count by 12's from 8 to 140, and back from 140 to 8.
6. Count by 12's from 7 to 139, and back from 139 to 7.
7. Count by 12's from 6 to 138, and back from 138 to 6.
8. Count by 12's from 5 to 137, and back from 137 to 5.
9. Count by 12's from 4 to 136, and back from 136 to 4.
10. Count by 12's from 3 to 135, and back from 135 to 3.
11. Count by 12's from 2 to 134, and back from 134 to 2.
12. Count by 12's from 1 to 133, and back from 133 to 1.
13. Find the cost of 2 bushels of potatoes at 20 cents a peck.
14. Find the cost of 5 gallons of oil at 5 cents a pint.
15. Find the cost of $2\frac{1}{2}$ pounds of beans at 8 cents a pound.
16. If a dozen lemons cost 15 cents, what will $2\frac{1}{3}$ dozen cost?
17. If a dozen lemons cost 15 cents, what will 4 lemons cost?
18. If a dozen lemons cost 15 cents, what will 16 lemons cost?
19. If 3 qts. of berries cost 15 cents, what will 5 qts. cost?
20. If a quart of milk costs 10 cents, what will one half of a gallon cost?
21. If a barrel of apples costs $2, how many barrels will $9 buy?
22. If 9 men can do a piece of work in 6 days, in what time can 3 men do the same work? In what time can 6 men do it?
23. If fruit costs 12 cents for half a pound, what will $2\frac{1}{3}$ pounds cost?
24. A 2-gallon jug filled with water was thrown over, and three quarts were spilled. How many qts. remained in the jug?
25. What will be the cost of 3 dozen eggs at the rate of 10 cents for a half-dozen?
26. How change a dollar when selling $3\frac{1}{4}$ pounds of beef at 20 cents a pound?
27. How many square inches on the top of a shelf $1\frac{1}{2}$ feet long and 6 inches wide?

1. Define *quantity*, and *unit*. (See page 7.)
2. Define *arithmetic*, *figure*, and *digit*.
3. Define *integer*, *fraction*, *numerator*, and *denominator*.
4. Define *number*, *abstract number*, and *concrete number*.
5. Define *notation*, and *numeration*. (See page 8.)
6. Explain the meaning of the *simple value* and *local value* of a figure.
7. Explain the change in the value of a figure when it is written in different orders.
8. Explain the use of *naught* or *cipher*.
9. Explain the several orders of the numeration table.
10. Explain the meaning of the *decimal system* of notation.
11. Explain the value of orders at the right of *units'* order.
12. Explain how a *decimal* is changed into a *common* fraction.
13. Explain how to use the word "*and*" in reading mixed decimals.
14. Explain how to multiply and divide by 10, 100, etc.
15. Explain the method of Roman notation.
16. Give the several kinds of denominate units.
17. Give the table for United States currency.
18. Change 100 francs to United States currency.
19. Change 100 marks to United States currency.
20. Give the table for English currency.
21. Change 10 shillings to United States currency.
22. Change 30 shillings to United States currency.
23. Give the table for Dry Measure.
24. Give the table for Liquid Measure.
25. How many pints in a peck? In $1\frac{1}{2}$ pecks?
26. How many quarts in a bushel? In $1\frac{1}{2}$ bushels?
27. How many pecks in 32 quarts? In 32 pints?
28. How many bushels in 64 pecks? In 64 quarts?
29. How many gills in a quart? In $1\frac{1}{2}$ quarts?
30. How many pints in a gallon? In $1\frac{1}{2}$ gallons?
31. How many pints in 32 gills? In 48 gills?
32. How many gallons in 32 pints? In 32 quarts?

1. Define *line, surface, solid.*
2. Give the table for Linear Measure.
3. How many inches in $5\frac{1}{2}$ feet? In $1\frac{1}{2}$ yards?
4. How many yards in 30 feet? In 4 yards?
5. How many feet in 4 rods? In $1\frac{1}{2}$ rods?
6. How many rods in $1\frac{1}{2}$ miles? In $1\frac{3}{8}$ miles?
7. How many common miles in 2 knots? In 6 knots?
8. How many feet in $10\frac{1}{2}$ fathoms? In 2 cables' lengths?
9. How many inches in 15 hands? How many feet?
10. Repeat the table for Square Measure.
11. How many square inches in $\frac{1}{2}$ square foot? In $\frac{1}{3}$? In $\frac{1}{4}$?
12. Give two ways to cut off one third of a square foot.
13. Give three ways to cut off one fourth of a square foot.
14. Give three ways to cut off one sixth of a square foot.
15. Give three forms in which an acre may be measured.
16. How many acres in a section? In $\frac{1}{2}$ section? In $\frac{1}{4}$ sec.
17. What part of a section are 160 A.? 80 A.? 40 A.? 10A.?
18. What part of an acre is a square chain? Are 2 sq. ch.?
19. What part of a cubic yard are 3 cubic ft.? Are 9 cu. ft.?
20. What part of a cord is a 4-foot cube?
21. How many ounces in $1\frac{1}{2}$ pounds? In $1\frac{3}{4}$ pounds?
22. How many pounds in $1\frac{1}{2}$ T.? In $\frac{1}{4}$ T.? In $\frac{1}{5}$ T.? In $\frac{1}{10}$ T.
23. How many pounds in $1\frac{1}{2}$ bu. wheat? Of corn? Of oats?
24. Repeat the table for Troy Weight.
25. How many grains in one half pint? In one half ounce?
26. How many pints in one half oz.? In one half pound?
27. Repeat the table for Apothecaries' Weight.
28. How many grains in $\frac{1}{2}$ ʒ? In $\frac{1}{2}$ ℥?
29. Repeat the table for Time Measure.
30. How many minutes in one half hour? In one half day?
31. How many days in the spring mos.? In the autumn mos.?
32. By what rule is a leap year determined?
33. How many units in four dozen? In four score?
34. How many degrees in a circle? In a quadrant?
35. Explain how the sun moves 15 degrees in 1 hour of time.

ADDITION.

ADDITION is the process of finding how many units there are in two or more numbers considered together; or, it is the process of finding a number that equals two or more numbers considered together.

Addends are the numbers that are added.

The *sum*, or *amount*, is the number obtained by addition.

PRINCIPLE—Only like numbers, and units of the same order, can be added.

1. Find the sum of 827, 948, and 759.

SOLUTION.

827
948
759
2534

Write the numbers so that units of the same order are in the same column. Add the column of units first. The sum is 24 units, or two tens and four units. Write the four units below the units' column and add the two tens to the next column. The sum of the next column is 13 tens, or three tens and one hundred. Write the three tens under the tens' column and add the one hundred to the next column. The sum of the hundreds is 25, which is written below to make the entire sum, 2,534.

2. Find the sum of 97, 84, 38, and 63.

SOLUTION.

97
84
38
63
282

Accountants often add two or more columns at once. Thus, 63 and 30 are 93, and 8 are 101, and 80 are 181, and 4 are 185, and 90 are 275, and 7 are 282. By omitting the names of the several sums, the process is read: 63, 93, 101, 181, 185, 275, 282. Sufficient practice will shorten the work still more by reading thus: 63, 101, 185, 282.

How write numbers to add them?

What column is first added?

Find the sum of each column, adding two columns at a time:

1.	2.	3.	4.
$23.00	$58.00	$45.47	$37.75
16.45	9.15	27.65	50.50
.75	33.65	16.80	25.75
5.95	55.85	72.45	62.15
.35	19.60	15.95	26.85
46.15	1.15	80.10	10.10
3.25	31.10	18.80	88.60
16.05	78.00	59.40	40.50
11.65	30.80	30.80	55.55
72.85	72.53	73.54	73.56
8.42	70.06	27.12	6.63
13.13	14.14	17.18	18.27
20.06	18.18	22.33	91.72
4.44	21.82	66.67	10.33
7.56	8.18	9.99	7.67

5.	6.	7.	8.
$12.25	$71.38	$36.75	$17.50
2.37	23.62	63.25	82.50
73.13	19.16	27.00	49.37
1.25	51.84	18.90	15.62
37.50	24.10	32.15	81.11
26.14	66.55	60.05	26.35
15.76	62.35	11.80	19.17
26.09	35.86	34.27	55.83
62.91	53.14	77.73	66.18
9.09	8.01	22.09	8.32
11.01	18.29	55.21	43.71
88.00	28.20	88.50	81.29
1.18	3.28	4.38	5.48
71.62	23.22	24.12	25.02
75.11	11.05	10.00	32.08

ADDITION OF COMPOUND NUMBERS is the operation of finding the sum of two or more compound numbers. The principles governing the process are the same as in simple addition.

The process is like that of simple numbers, but instead of the scale of 10, as in simple numbers, the scale varies with the denomination. (See the tables for denominate numbers.)

1. Find the sum of 13 gal. 2 qt. 1 pt. 3 gi.; 14 gal. 2 qt. 2 gi.; 7 gal. 3 qt. 3 gi.; 9 gal. 1 qt. 1 pt. 2 gi.; 6 qt. 1 pt. 1 gi.

SOLUTION.

gal.	qt.	pt.	gi.
13	2	1	3
14	2	0	2
7	3	0	3
9	1	1	2
	6	1	1
47	0	1	3

Numbers of the same denomination are written in the same column. The sum of the column of gills is 11 gills, or 2 pt. 3 gills. The 3 gills are written under the column added, and the 2 pints are added with the column of pints. The sum of the column of pints is 5 pints, or 2 qt. 1 pt. The 1 pint is written under the column of pints, and the 2 quarts are added to the column of quarts, and so on until all the columns are added.

2. Find the sum of 13 cwt. 51 lbs. 13 oz.; 3 cwt. 18 lbs. 9 oz.; 25 cwt. 31 lbs. 15 oz.

3. Find the sum of 58 gal. 1 qt. 1 pt. 3 gi.; 45 gal. 3 qt. 1 pt. 1 gi.; 38 gal. 1 qt. 1 pt. 3 gi.; 26 gal. 3 qt. 3 gi.

4. Find the sum of 9 cu. yd. 13 cu. ft. 1556 cu. in.; 86 cu. yd. 22 cu. ft. 695 cu. in.; 34 cu. yd. 8 cu. ft. 924 cu. in.

5. Find the sum of 7 cwt. 97 lbs. 13 ozs.; 7 T. 8 cwt. 7 lbs. 7 oz.; 11 T. 11 cwt. 11 lb. 11 oz.; 179 cwt. 1780 lbs. 9 oz.; 137 T. 19 cwt. 89 lbs. 15 oz.

6. Find the sum of 150 sq. yd. 2 sq. ft. 27 cu. in.; 55 sq. yd. 9 sq. ft. 14 sq. in.; 130 sq. yd. 7 sq. ft. 125 sq. in.

7. Find the sum of 75 bu. 2 pk. 7 qt.; 25 bu. 3 pk. 6 qt.; 60 bu. 1 pk. 4 qt.; 27 bu. 2 pk. 5 qt.

8. Find the sum of 22 rd. 2 yd. 2 ft.; 9 rd. 4 yd. 2 ft.; 16 rd. 3 yd. 1 ft.; 16 rd. 4 yd. 2 ft.

1.

lb.	oz.	pwt.	gr.
6	5	4	1
2	2	13	19
	4	0	6
8	9	9	4
3	3	3	12

2.

lb.	oz.	pwt.	gr.
7	3	0	5
11	2	17	22
20	5	11	20
7	10	13	13
	7	17	14

3.

lb.	℥	ʒ	℈	gr.
8	10	7	2	19
10	0	6	0	10
	1	2	1	15
6	2	6	1	12
9	0	6	2	14

4.

lb.	℥	ʒ	℈
2	11	6	0
10	8	3	1
14	10	2	2
	6	5	1
10	6	5	2

5.

bbl.	gal.	qt.
10	20	2
8	30	3
2	5	1
22	0	3
6	15	1

6.

bu.	pk.	qt.	pt.
20	3	7	1
12	2	3	1
20	2	0	1
10	2	5	0
9	0	5	2

7.

bu.	pk.	qt.	pt.
10	1	1	1
16	2	3	0
5	2	4	1
8	0	0	1
15	2	2	0

8.

da.	hr.	min.
15	18	50
24	12	45
4	0	40
10	5	45
2	15	10

9.

lb.	oz.	pwt.
175	11	16
68	9	15
242	6	13
125	7	9
93	9	19

10.

lb.	℥	ʒ
13	9	5
47	11	7
50	9	4
39	3	6
47	4	7

11.

lb.	oz.	dr.
13	14	15
25	8	7
48	5	9
74	7	4
18	12	13

12.

gal.	qt.	pt.
15	3	1
64	0	1
69	1	1
65	3	0
19	1	1

13.

rd.	yd.	ft.	in.
12	4	2	6
34	2	1	10
8	1	0	20
36	2	2	9
	5	1	6

SUBTRACTION is the process of finding the difference between two numbers.

The *subtrahend* is the number subtracted.

The *minuend* is the number from which the subtrahend is subtracted.

The *number* or *difference* is the number left after subtraction.

PRINCIPLES.

Only like numbers and units of the same order can be subtracted.

The sum of the difference and the subtrahend must equal the minuend.

1. What is the difference between 5,283 and 1,468?

SOLUTION.

4	12	7	13=Minuend, changed in form.
.....	.....		
5	2	8	3=Minuend.
1	4	6	8=Subtrahend.
3	8	1	5=Remainder.

Begin at the right hand. Since 8 units cannot be subtracted from 3 units, reduce 1 of the 8 tens of the minuend to units; 1 ten=10 units, which, added to the 3 units, make 13 units. 8 units from 13 units leave 5 units. 6 tens from the 7 tens left in the minuend leave 1 ten. Since 4 hundreds cannot be subtracted from 2 hundreds, reduce 1 of the 5 thousands to hundreds; 1 thousand=10 hundreds, which, added to the 2 hundreds, make 12 hundreds. 4 hundreds from 12 hundreds leave 8 hundreds, 1 thousand from the 4 thousands left in the minuend leaves 3 thousands.

The remainder is 3 thousands, 8 hundreds, 1 ten, and 5 units; or, 3,815.

2. What is the difference between 4,003 and 2,715?

3. What is the difference between 3,333 and 2,715?

Subtract—

1.	2.	3.	4.	5.	6.
34387	50381	61874	22820	61854	61235
21857	27596	18027	17232	32108	41387

7.	8.	9.	10.	11.	12.
42106	81567	87554	77376	52382	34567
37368	27876	57843	51847	36875	19101

13.	14.	15.	16.
78287	63529	52356	42131
18534	17152	42873	24516

Find the amount in the bank to the credit of the following accounts—

17. Deposit, $ 75.16; checks, $ 15.25, $ 13.25, $ 14.15, $ 9.60

18. Deposit, $ 91.25; checks, $ 37.20, $ 23.15, $ 17.35, $ 9.45

19. Deposit, $785.00; checks, $150.75, $231.90, $ 5.85, $ 9.75

20. Deposit, $927.00; checks, $ 63.71, $ 87.18, $122.42, $53.58

21. Deposit, $175.00; checks, $ 26.15, $ 11.48, $ 1.15, $19.36

22. Deposit, $ 35.18; checks, $ 13.75, $ 1.10, $ 9.05, $ 9.20

23. Deposit, $150.00; checks, $ 47.51, $ 30.25, $ 52.62, $ 7.65

24. Albert and Henry each had $15.65. Albert gave Henry $6.85; how much did each one then have, and how much more did Henry have than Albert.

25. Edward has $675, George has $813, and William has as many dollars less than Edward, as Edward has less than George. How many dollars has William?

26. Willie had $8.37, and spent $1.98, and still had $2.37 more than Edwin. How much had Edwin?

27. A horse cost $175, a carriage cost $230, and a set of harness cost as much less than the horse as the carriage cost more than the horse. Find the cost of the harness.

SUBTRACTION OF COMPOUND NUMBERS is the operation of finding the difference between two compound numbers. The principles are the same as in subtraction of simple numbers.

1. Subtract 8 lb. 10 oz. 18 pwt. 21 gr. from 12 lb. 3 oz. 16 pwt. 14 gr.

SOLUTION.

11	14	35	38=Minuend changed.
lb.	oz.	pwt.	gr.
12	3	16	14=Minuend.
8	10	18	21=Subtrahend.
3	4	17	17=Remainder.

Since 21 gr. cannot be subtracted from 14 gr., reduce one of the 16 pwt. to grains. 1 pwt.=24 gr., which, added to the 14 gr., give 38 gr., from which subtracting 21 gr., leave 17 gr.

Since 18 pwt. cannot be subtracted from the 15 pwt., (left after the reduction) reduce 1 of the 3 oz. to pennyweights. 1 oz.= 20 pwt., which, added to the 15 pwt., give 35 pwt., from which subtracting 18 pwt., leave 17 pwt.

Since 10 oz. cannot be subtracted from the 2 oz. (left after the reduction), reduce 1 lb. to ounces, etc.

The entire remainder is 3 lb. 4 oz. 17 pwt. 17 gr.

2. Subtract 14 rd. 4 yd. 2 ft. 8 in., from 28 rd. 3 yd. 0 ft. 4 in.

SOLUTION.

27	8	1	10=Minuend changed.
rd.	yd.	ft.	in.
28	3	0	4=Minuend.
14	4	2	8=Subtrahend.
13	3	2	2=Remainder.

The subtraction requires that 1 rod shall be reduced. 1 rd.= $5\frac{1}{2}$ yd.; adding the 5 yd. to the 3 yd., give 8 yd.; $\frac{1}{2}$ yd.=1 ft. 6 in., which, added to the 0 ft. 4 in., give 1 ft. 10 in. Now, subtracting 8 in. from 10 in., leave 2 in., etc.

TO FIND THE TIME BETWEEN TWO DATES.

1. What time elapsed from 2 o'clock P. M. September 4, 1892, to 10 o'clock A. M., April 15, 1900?

SOLUTION.

yr.	mo.	da.	hr.
1900	4	15	10
1892	9	4	14
7	7	10	20

Write the latter date for the *minuend*, and the earlier for the *subtrahend*, giving the month its *number* instead of the *name*. Reckon 12 mo. as a year and 30 da. a month. But to be *exact*, the *true* number of days in each month and parts of a month must be reckoned when the time is *less than a year*. When hours are to be obtained, reckon from 12 o'clock, midnight.

Find the time between—

2. The discovery of America by Columbus, October 12, 1492, and the admission of California as a state, September 9, 1850.

3. The founding of Jamestown, Va., May 13, 1607, and the admission of Washington as a state, February 22, 1889.

4. The landing of the Pilgrims at Plymouth, Mass., December 4, 1620, and the Declaration of Independence, July 4, 1776.

5. The "Boston Massacre," March 5, 1770, and the first bloodshed of the Civil War, April 19, 1861.

6. The inauguration of the first president, April 30, 1789, and the death of President Garfield, September 19, 1881.

7. The battle of New Orleans, January 8, 1815, and the battle of Bull Run, July 21, 1861.

8. The death of Adams and Jefferson, July 4, 1826, and the death of Abraham Lincoln, April 15, 1865.

9. The battle of Bunker Hill, June 17, 1775, and the death of Gen. Grant, July 23, 1885.

10. The adoption of the "Stars and Stripes," by Congress, June 14, 1777, and Dewey's victory at Manila, May 1, 1898.

11. November 3, 1794, and June 12, 1878.

12. May 25, 1803, and April 27, 1882.

MULTIPLICATION is the process of taking one number as many times as there are units in another.

The *multiplicand* is the number taken or multiplied.

The *multiplier* is the number that shows how many times the multiplicand is taken.

The *product* is the result obtained by multiplication.

PRINCIPLES.

The multiplier must be an abstract number.

The multiplicand and the product are like numbers.

The product is the same in whatever order the numbers are multiplied.

1. Multiply 24 by 14.

$24\times 4 = 96$
$24\times 10 = 240$
$24\times 14 = 336$

$$\begin{array}{r} 24 \\ 14 \\ \hline 96 \\ 24 \\ \hline 336 \end{array}$$

2. Multiply 4736 by 2384.

$$\begin{array}{r} 4736 \\ 2384 \\ \hline 18944 = \text{4 times 4736} \\ 37888 = \text{80 times 4736} \\ 14208 = \text{300 times 4736} \\ 9472 = \text{2000 times 4736} \\ \hline 11290624 = \text{2384 times 4736} \end{array}$$

3. Multiply 435 by 12.

(*a*) $435 = 400 + 30 + 5$
$12 = 12 \quad 12 \quad 12$
$5220 = 4800 + 360 + 60$

(*b*) $435 = \text{Multiplicand.}$
$12 = \text{Multiplier.}$

$$\begin{array}{rl} 60 & \text{Units.} \\ 36 & \text{Tens.} \\ 48 & \text{Hundreds.} \\ \hline 5220 & \end{array}$$

4. Multiply the sum of 398 and 467 by their difference.

IN MULTIPLICATION OF DECIMALS the process is the same as in multiplication of integers, except in placing the *decimal point.*

1. Multiply .25 by 9.

SOLUTION.

.25
9
2.25

Multiply as in integers. The principle requiring the product to be of the same denomination as the multiplicand determines the position of the decimal point. Since the multiplicand is 25 *hundredths*, the product must be *hundredths;* and 225 *hundredths* equal 2 units and 25 *hundredths*, which is written 2.25. Two decimal places are in the multiplicand, and as many are pointed off in the product.

2. Multiply .25 by .9.

SOLUTION.

.25
.9
.225

If the multiplier were integer 9, the product would be 2.25; but since .9 is one tenth as large as 9, the product must be one tenth as large as 2.25, which is found by moving the decimal point one place to the left. As many decimal places are pointed off in the product as are in both multiplier and multiplicand.

3. Multiply .25 by .09.

SOLUTION.

.25
.09
.0225

If the multiplier were the integer 9, the product would be 2.25; but since .09 is one hundredth of 9, the product must be one hundredth of 2.25, which is found by moving the decimal point two places to the left. As there are but three figures in the product, it is necessary to prefix a cipher.

4. Find the product of 2.5 and 4.25
5. Find the product of 35 and 31.6
6. Find the product of 34 and 7.34
7. Find the product of .75 and 58.7
8. Find the product of 9.02 and 9.02
9. Find the product of 7.26 and 72.6
10. Find the product of 135 and 82.54
11. Find the product of 2.46 and 325.7

A COMPOSITE NUMBER is one produced by multiplying two or more numbers together. Thus, 35 is a composite number, since it may be produced by multiplying together 5 and 7. The 5 and 7 are called the *factors* of 35.

1. Multiply 48 by 35.

SOLUTION.

48=Multiplicand.
5=First Factor.
240
7=Second Factor.
1680=Product.

The most important application of the foregoing principle is made when the multiplier contains 10, 100, 1000, or some other power of 10 as a factor.

To multiply by 800, or 8 times 100, first multiply by 100 by removing the decimal point two places to the right and then multiply that product by 8; or, simply multiply by 8 and remove the point two places further to the right.

To multiply by 7000, multiply by 7 and remove the point three places to the right.

2. What is the product of 694.7×4000?

SOLUTION.

694.7
4000
2778800.

Since 4000=4 times 1000, we may find the required product by multiplying 694.7 by 4, and removing the point three places to the right.

Find the following products—

3. 5794×20.
4. 36.85×700.
5. 4.396×40.
6. 5764×3000.
7. 8342×500.
8. 8427×60.
9. $.0049 \times 3000$.
10. 73648×60.
11. 4957.3×300.
12. 2796×8000.
13. 356.7×400.
14. $.079 \times 5000$.
15. 81.23×9000.
16. $.0547 \times 80000$.
17. 52.78×500.
18. 327.1×9000.
19. 4358×600.
20. $.329 \times 5000$.
21. 6275×40000.
22. 5279×300000.

A POWER OF A NUMBER is any product which arises from multiplying the number continually by itself.

THE ROOT, or simple factor, is called the *first power*.

THE SECOND POWER is the product of the root by itself.

THE THIRD POWER is the product, when the root is taken 3 times as a factor.

THE FOURTH POWER is the product, when it is taken 4 times.

THE FIFTH POWER is the product, when it is taken 5 times.

The number denoting how many times the root is taken as a factor, is called the *exponent* of the power. It is written a little at the right and over the root. Thus, if the equal factor or root is 3,

$3^1 = 3$, the first power, or root.
$3^2 = 3 \times 3 = 9$, the second power, or square.
$3^3 = 3 \times 3 \times 3 = 27$, the third power, or cube.
$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the fourth power.
$3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$, the fifth power.

INVOLUTION is the process of finding the powers of numbers.

In finding any power, one multiplication gives the second power; hence, *the number of multiplications is 1 less than the exponent.*

1. What is the third power of 16?

SOLUTION.

$16 \times 16 \times 16 =$ .....

2. Find the square of each of the digits.
3. Find the cube of each of the digits.
4. Find the third power of 50.
5. Find the fifth power of 40.
6. Find the product of 4^3 times 2^4.
7. Find the product of 5^4 times 4^2.
8. Find the sum of 2.5^2 and 1.5^2.
9. Show by a diagram or by blocks that $3^2 = 9$.
10. Show by a diagram or by blocks that $3^3 = 27$.
11. Show by a diagram or by blocks that $12^2 = 144$.

MULTIPLICATION OF COMPOUND NUMBERS is the operation of taking a compound number as many times as there are units in the multiplier.

The principles are the same as in the multiplication of simple numbers.

1. Multiply 11 bu. 3 pks. 2 qts. by 7.

SOLUTION.

11 bu. 3 pks. 2 qts.	7×2 qts.=14 qts.=1 pk. and 6 qts.
7	7×3 pks.=21 pks.
82 bu. 2 pks. 6 qts.	21 pks.+1 pk.=22 pks.=5 bu. and 2 pks.
	7×11 bu.=77 bu.
	77 bu.+5 bu.=82 bu.

The complete product is 82 bushels, 2 pecks, and 6 quarts.

2. Multiply 18 T. 2 cwt. 16 lb. 9 oz. by 48.

3. Multiply 5 yr. 8 mo. 2 wk. 3 da. 42 min. by 56.

4. Multiply 7 gal. 3 qts. 1 pt. 3 gi. by 36.

5. Multiply 7 wks. 4 da. 13 hrs. 27 min. 36 sec. by 9.

6. Multiply 23 cu. yd. 6 cu. ft. 59 cu in. by 8.

7. Multiply 6 lbs. 8 oz. 15 pwt. 19 gr. by 7.

8. Multiply 1 T. 3 cwt. 3 lbs. 9 oz. by 100.

9. Multiply 17 hrs. 47 min. 15 sec. by 25.

10. Find the amount of the following bill—

15 yards of broadcloth,	at	£ 1	3 s. 6 d.	per yard.
12 yards of silk,	at	18	3	per yard.
20 yards of calico,	at	1	9	per yard.
24 yards of sheeting,	at	1	3	per yard.
22 yards of muslin,	at	3	4	per yard.

11. Considering the year to consist accurately of 365 days, 5 hours, 48 minutes, 48 seconds, find the exact length of 43 years.

12. How long will it take a boy to saw 10 cords of wood, if he takes 7 hours, 30 minutes, 45 seconds to saw one cord, allowing 10 working hours for each day?

13. A tank, when filled, contains 1 bl. 10 gal. 3 qt. 1 pt. of oil. How much will 4 such tanks contain?

REDUCTION is the operation of changing a number from one unit to another, without altering its value.

Reduction descending is the operation of changing a number from a greater unit to a less.

Reduction ascending is the operation of changing a number from a less unit to a greater.

If we have 4 yards, in which the unit is 1 yard, and wish to change to feet, the units of the scale will be 3, since 3 feet make 1 yard; therefore, the number of feet will be—

$$4 \times 3 \text{ ft.} = 12 \text{ ft.}$$

1. Reduce 46 bu. 2 pk. 5 qt. to quarts.

SOLUTION.

1 bushel = 4 pecks.
46 bushels = 46×4 pecks, or 184 pecks.
184 pecks + 2 pecks=186 pecks.
1 peck = 8 quarts.
186 pecks =186×8 quarts, or 1488 quarts.
1488 quarts + 5 quarts=1493 quarts.

Hence, 46 bu. 2 pk. 5 qt.=1493 quarts.

NOTE—It is seen by this solution that the process of reduction descending consists of multiplication and addition.

2. In 4 lb. 6 oz. 13 pwt. 5 gr. are how many grains?

3. In 3 acres 27 square rods are how many square feet?

4. In 96 sq. yd. 5 sq. ft. are how many square feet?

5. In 46 bu. 3 pk. 7 qt. 1 pt. are how many pints?

6. In 42 ℥ 6 ʒ 2 ℈ are how scruples?

7. In 287 leagues 1 mi. 3 fur. are how many rods?

8. What will 15 gal. of wine cost at 5 s. 3½ d. per gallon?

9. How much will 14 bu. 2 pk. 5 qt. 1 pt. of nuts cost at 3 cents per pint?

10. How much will it cost to build a wall 1 mi. 1 fur. in length, at $62.50 per furlong?

11. Find the cost of digging a ditch 20 rd. 4 yd. 2 ft. in length, at 60 cents a linear foot.

DIVISION.

DIVISION is the process of finding how many times one number is contained in another; or, of finding one of the equal parts of a number.

The *dividend* is the number divided.

The *divisor* is the number contained in the dividend.

The *quotient* is the result obtained by division.

When the dividend does not contain the divisor an exact number of times, the part of the dividend left is called the remainder. It must be less than the divisor.

When there is no remainder, the division is said to be *exact*.

PRINCIPLES.

When the divisor and the dividend are like numbers, the quotient is an abstract number.

When the divisor is an abstract number, the dividend and the quotient are like numbers.

The product of the divisor and the quotient, plus the remainder, if any, is equal to the dividend.

1. Divide 17668 by 7.

SOLUTION I—SHORT DIVISION.

$$\begin{array}{r} 14000\div 7=2000 \\ 3500\div 7=\ 500 \\ 140\div 7=\ \ 20 \\ 28\div 7=\ \ \ 4 \\ \hline 17668\div 7=2524 \end{array} \qquad \begin{array}{r} 7)17668 \\ \hline 2524 \end{array}$$

2. Divide 8970 by 26.

SOLUTION II—LONG DIVISION.

$$\begin{array}{rrrr} & 26)8970(& 300 & \\ 300\times 26= & 7800 & 40 & \\ & \overline{1170} & 5 & \\ 40\times 26= & 1040 & \overline{345} & \\ & \overline{130} & & \\ 5\times 26= & 130 & & \\ & \underline{\ \ \ } & & \end{array} \qquad \begin{array}{r} 345 \\ 26)8970 \\ 78 \\ \overline{117} \\ 104 \\ \overline{130} \\ 130 \\ \underline{\ \ \ } \end{array}$$

Find the quotients—

1.	239÷43	**18.**	4258÷ 46	**35.**	36854÷ 59
2.	627÷81	**19.**	2397÷ 68	**36.**	87943÷ 45
3.	518÷64	**20.**	6754÷ 83	**37.**	62379÷ 83
4.	697÷79	**21.**	3586÷412	**38.**	25978÷ 127
5.	416÷68	**22.**	3847÷212	**39.**	36594÷ 41
6.	213÷47	**23.**	5978÷613	**40.**	63974÷ 25
7.	438÷75	**24.**	2974÷391	**41.**	39547÷ 62
8.	216÷45	**25.**	6137÷196	**42.**	54296÷ 83
9.	715÷92	**26.**	3817÷ 61	**43.**	47698÷ 45
10.	317÷53	**27.**	1358÷ 42	**44.**	63897÷ 328
11.	2375÷52	**28.**	3157÷ 83	**45.**	159468÷1274
12.	3796÷43	**29.**	3773÷ 93	**46.**	294875÷3079
13.	4986÷72	**30.**	6179÷ 87	**47.**	205879÷4983
14.	5389÷91	**31.**	3579÷ 49	**48.**	62574÷ 99
15.	2456÷33	**32.**	3163÷ 36	**49.**	379856÷5387
16.	4795÷49	**33.**	43796÷ 82	**50.**	625794÷8215
17.	1396÷37	**34.**	25794÷ 71	**51.**	208598÷2054

52. How many months will it take a man who earns $90 per month, to earn $2,745?

53. A man received $1,363 for cattle, at $29 per head. How many cattle did he sell?

54. How may suits of clothes, at $32 each, can be bought for $1,024?

55. At the rate of 19 bushels of wheat to an acre, how many acres will produce 1,294 bushels?

56. A farm of 550 acres was sold for $27,500. What was the price per acre?

57. A person having a salary of $3,400, has $1,940 at the end of the year. How much were his average daily expenses, if we count 365 days to the year?

58. A person bought two farms, one of 97 acres, at $51 per acre, the other of 111 acres, at $47 per acre. He paid $9,539 cash, and for the balance he gave 5 horses. What was the value of each horse?

Since the dividend is equal to the product of the divisor and quotient, it must contain as many decimal places as there are decimals in both divisor and quotient. Therefore, in division of decimals there must be as many decimals in the quotient as the decimal places in the dividend *exceed* those in the divisor.

1. Divide .125 by 25.

SOLUTION.

25).125(.005
125

Divide as with integers. The dividend has *three* decimal places, the divisor has *none*. The quotient must therefore have *three*. Ciphers are prefixed to fill the vacant orders.

2. Divide 12.5 by .25.

SOLUTION.

.25)12.50(50
12 5
0

When the divisor has *more* decimals than the dividend, annex ciphers to the dividend until its decimal places *equal* those of the divisor; the quotient will be a whole number.

3. Divide 332.4 by 16.

SOLUTION.

16)332.400(20.775
32
124
112
120
112
80
80

After dividing all the figures of the dividend, if there be a remainder, annex ciphers to it, and continue the division till there is no remainder, or until the quotient is sufficiently exact. In placing the point in the quotient, regard the ciphers annexed as decimal places.

4. Divide 144 by 3.6
5. Divide 14.4 by .36
6. Divide 1.44 by 36
7. Divide .144 by 3.6
8. Divide 264 by .033
9. Divide .264 by 240
10. Divide 288 by .036
11. Divide 453 by .015
12. Divide 18 by 900
13. Divide 785.4 by 700
14. Divide 12.5 by 800
15. Divide 31.75 by .025
16. Divide 15.336 by 2700
17. Divide 843.66 by 1800

DIVISION OF COMPOUND NUMBERS is the operation of dividing a compound number by an abstract number, or by a similar denominate number.

The principles used are the same as those used in the division of simple numbers.

1. Divide 42 bushels 3 pecks 4 quarts 1 pint by 5.

SOLUTION.

bu.	pks.	qts.	pt.	
5)42	3	4	1	⅕ of 42 bu.=8 bu. and 2 bu. remaining.
8	2	2	1	2 bu.=8 pks.
				8 pks+3 pks.=11 pks.
				⅕ of 11 pks.=2 pks. and 1 pk. remaining.
				1 pk.=8 qts.
				8 qts.+4 qts.=12 qts.
				⅕ of 12 qts.=2 qts. and 2 qts. remaining.
				2 qts.=4 pts.
				4 pts.+1 pt.=5 pts.
				⅕ of 5 pts.=1 pt.

The complete quotient is 8 bushels 2 pecks 2 quarts 1 pint.

2. Divide 24 days 19 hours 30 minutes by 6.

3. Divide 43 weeks 2 days 9 hours by 9.

4. Divide 39 bushels 2 pecks 1 quart by 11.

5. Divide 1 ton 17 hundredweight 62 pounds by 7.

6. Divide 56 pounds 6 ounces 17 pennyweight by 9.

7. Divide 14 hundredweight 38 pounds by 13.

8. Divide 21 weeks 5 days by 3 weeks 5 days.

9. Divide 62 pounds 9 ounces by 5 pounds 11 ounces.

10. Divide 19 bushels 3 pecks 3 quarts by 3 bu. 3 pks. 7 qts.

NOTE—**Reduce both quantities to the same denomination.**

11. A wood cutter cut 60 cords of wood in 24 days. How much did he cut per day?

12. From 28 acres were harvested 976 bushels 2 pecks of wheat. What was the yield per acre?

13. A steamboat went 40 miles in $3\frac{1}{2}$ hours. What was the rate per hour?

1. Reduce 5,433 pints to higher denominations.

SOLUTION.

2)5433 pt. —1 pt.
8)2716 qt. —4 qt.
4)339 pk.—3 pk.
84 bu.

Since 2 pints make 1 quart, there will be $\frac{1}{2}$ as many quarts as there are pints, which is found by dividing 5,433 by 2. This makes 2,716 quarts, with 1 pint remaining.

Since 8 quarts make 1 peck, there will be $\frac{1}{8}$ as many pecks as there are quarts, which is found by dividing 2,716 by 8. This makes 339 pecks, with 4 quarts remaining.

Since 4 pecks make 1 bushel, there will be $\frac{1}{4}$ as many bushels as there are pecks, which is found by dividing 339 by 4.

This gives 84 bushels, with 3 pecks remaining.

Hence, 5,433 pints equal 84 bushels 3 pecks 4 quarts 1 pint.

It is seen from this solution that this process of reduction is chiefly one of division.

2. Reduce 12530 grains to pounds.
3. Reduce 805 pennyweights to pounds.
4. Reduce 30941 grains to pounds.
5. Reduce 29239 grains to pounds.
6. Reduce 133 inches to yards.
7. Reduce 181 inches to yards.
8. Reduce 2240 rods to miles.
9. Reduce 2200 rods to miles.
10. Reduce 243 square rods to acres.
11. Reduce 603 square rods to acres.
12. Reduce 4176 square inches to square yards.
13. Reduce 4323 minutes to days.
14. Reduce 20280 minutes to weeks.
15. Reduce 41761 minutes to months.
16. Reduce 937467 drams (Av.) to higher denominations.
17. Reduce 623458 grains (Troy) to higher denominations.
18. Reduce 3517965 square inches to higher denominations.
19. Reduce 527943 cubic inches to higher denominations.
20. A person bought 1,224 yards of cloth, in pieces each containing 34 yards. How many pieces did he buy?

An *account* is a record of business transactions.

The *debit* side of an account shows the items to be paid to the accountant.

The *credit* side of an account shows the items to be paid by the accountant.

A *bill* is a detailed statement of goods sold, of services rendered, or money paid, with the dates, prices, and amount.

A *debt* is what is due from one person to another.

A *debtor* is a person from whom a debt is due.

A *creditor* is a person to whom a debt is due.

To *receipt* a bill is to write on the bill, usually at the bottom of the statement, the words "Received payment," or "Paid," followed by the creditor's name, or by the name of some person authorized to receipt the account.

A bill is extended by writing the amount of each item in a a column.

A bill is footed by finding the sum of the column extended.

Copy, extend, foot, and receipt the following bill :—

PORTLAND, OREGON, June 1, 1903.

MR. HAMILTON PLATT.

Bought of THE RAY GROCERY CO.

May 2	8 lbs. Java Coffee	@	35 ¢	
	15 lbs. Gran. Sugar	@	6½¢	
May 5	4 gal. Syrup	@	75 ¢	
	10 lbs. Rice	@	6¼¢	
May 12	5 cans Tomatoes	@	12½¢	
	6 cans Pineapples	@	25 ¢	
May 19	5 lbs. Cheese	@	25 ¢	
	2 lbs. Butter	@	35 ¢	
				$

Make an original bill of goods bought.

Is a bill of any value after it is paid?

1. Divide 35 times 99 by 63.

SOLUTION.

$$\frac{35\times99}{63}=\frac{5\times\cancel{7}\times\cancel{9}\times11}{\cancel{7}\times\cancel{9}}=5\times11=55$$

Separate the composite numbers into their prime factors. Dividing both dividend and divisor by the same number does not change the quotient. Dividing both by the common factors, 7 and 9, leaves 5×11, or 55, for a quotient. The process of taking out common factors is called *cancellation*, and is indicated by drawing a line through the common factors.

Solve by cancellation—

2. $\frac{15\times17\times24}{51\times48\times30}$

3. $\frac{21\times16\times32\times49}{24\times28\times35\times36}$

4. $\frac{17\times13\times36\times28}{38\times49\times51\times39}$

5. $\frac{10\times11\times12\times9\times8}{24\times27\times60\times33\times55}$

6. $\frac{15\times63\times12\times77}{33\times40\times56\times144}$

7. $\frac{144\times18\times75\times132}{150\times1728\times96\times38}$

8. $\frac{299\times77\times374\times262}{121\times713\times324\times561}$

9. $\frac{64\times27\times28\times90}{18\times56\times16\times320}$

10. $\frac{77\times76\times75\times114}{44\times38\times25\times231}$

11. $\frac{76\times82\times57\times76}{19\times19\times164\times228}$

12. $\frac{.15\times.25\times500}{5\times15\times.025}$

13. $\frac{2.1\times160\times32}{.8\times7\times16}$

14. $\frac{27\times6\times.5\times24}{.8\times3\times10\times.3}$

15. $\frac{1.2\times9\times1.1\times100}{4\times90\times.1\times.2}$

16. $\frac{7.7\times63\times.15\times120}{11\times.05\times20\times.21}$

17. $\frac{280\times2.7\times.9\times16}{.8\times3\times1.8\times14}$

18. $\frac{1.5\times75\times144\times.1}{150\times.75\times1.2\times120}$

19. $\frac{.9\times150\times64\times.1\times13.2}{180\times.3\times3.2\times.01\times264}$

20. $\frac{5\times3.6\times56\times16\times16.2}{.05\times12\times9\times11.2\times90}$

21. $\frac{21\times3.8\times16.4\times57\times820}{30\times.07\times.82\times190\times1.14}$

22. Write the rule for cancellation.

PROPERTIES OF NUMBERS.

AN EXACT DIVISOR of a number is any number, except 1 and the number itself, that will divide it without a remainder.

A number is *divisible* by another when there is no remainder after division.

A *factor* of a number is an exact divisor of the number.

An *even number* is one divisible by 2.

Other numbers are *odd*.

A *prime number* is one which has no exact divisor. The numbers 1, 2, 3, 5, 7, 11, 13, 17, etc , are prime numbers.

A *composite number* is one having exact divisors.

Two numbers are *prime to each other* when no number is an exact divisor of both.

A *common divisor* of two or more numbers is a number that will exactly divide each of them.

The *greatest common divisor (G. C. D.)* of two or more numbers is the greatest number that will exactly divide each of them.

A *multiple* is a number of which the given number is a factor.

A *common multiple* is a multiple of two or more numbers.

The *least common multiple (L. C. M.)* is the least multiple common to two or more numbers.

EXACT DIVISORS.

1. Two is an exact divisor of any even number.

2. Three is an exact divisor of a number the sum of whose digits is divisible by 3.

3. Four is an exact divisor of a number when it will exactly divide the number expressed by the two right-hand digits.

4. Five is an exact divisor of every number whose right-hand figure is 0 or 5.

5. Six is an exact divisor of an even number of which 3 is a factor.

6. Eight is an exact divisor of a number if it is a factor of the number expressed by the three right-hand figures of the number, or if the number ends in three ciphers.

1. Find the prime factors of 66.

SOLUTION.

2)66 Dividing 66 by a prime factor, the first quotient is 33.
3)33 Dividing this number by a prime factor, the second quotient is 11, which is also a prime number. The prime factors of 66 are 2, 3, and 11.
11

Find the prime factors of—

2.	144	**11.**	512	**20.**	1001	**29.**	5368
3.	132	**12.**	732	**21.**	1728	**30.**	8145
4.	169	**13.**	527	**22.**	2156	**31.**	8283
5.	205	**14.**	931	**23.**	3125	**32.**	1972
6.	896	**15.**	621	**24.**	6157	**33.**	2512
7.	275	**16.**	507	**25.**	1431	**34.**	2021
8.	141	**17.**	625	**26.**	8375	**35.**	2737
9.	222	**18.**	659	**27.**	6561	**36.**	7056
10.	341	**19.**	841	**28.**	2310	**37.**	6075

38. Find the G. C. D. of 18, 30, and 48.

SOLUTION.

$18=2\times3\times3$
$30=2\times3\times5$
$48=2\times3\times2\times2\times2$
$2\times3=6$. Ans.

Factoring all the numbers, it is found that 2 and 3 are common factors to the numbers, and the *only* common factors. Their product is the G. C. D.

In the above solution it is seen that—

30 equals 5 times 6.
48 equals 8 times 6.
Adding. 78 equals 13 times 6.

48 equals 8 times 6.
30 equals 5 times 6.
Substracting. 18 equals 3 times 6.

PRINCIPLES.

A common divisor of two numbers is a divisor of their sum, and also of their difference.

The greatest common divisor of two or more numbers is the product of their common prime factors.

1. What is the greatest common divisor of 3,473 and 8,909?

SOLUTION.

3473)8909(2
6946
1963)3473(1
1963
1510)1963(1
1510
453)1510(3
1359
151)453(3
453

As the factors of these numbers are not readily recognized, divide the greater by the less, which gives 2 for a quotient and 1,963 for a remainder. Now find the greatest common divisor of 1,963 and 3,473. Divide 3,473 by 1,963, which gives 1 for a quotient and 1,510 for a remainder, etc., until we find that 151 is the first exact divisor. It is the G. C. D.

Find the G. C. D.—

2. Of 365 and 511
3. Of 864 and 420
4. Of 775 and 1800
5. Of 2628 and 2484
6. Of 2268 and 3444
7. Of 14, 18, and 24
8. Of 837, 1134, 1347
9. Of 78, 52, 13, 416
10. Of 154 and 210
11. Of 316 and 664
12. Of 679 and 1869
13. Of 917 and 1495
14. Of 1313 and 4108
15. Of 1649 and 5423

16. Find the L. C. M. of 30, 42 and 66.

SOLUTION.

$30=2\times3\times5$
$42=2\times3\times7$
$66=2\times3\times11$
$2\times3\times11\times7\times5=2310$

The L. C. M. cannot be less than the largest number. It must therefore contain 66, and all the prime factors of 66, which are 2, 3, and 11. A multiple of 42 must likewise contain the factors 2, 3, and 7. Since 2 and 3 are used once, the 7 is taken as an additional factor with 2, 3, and 11. Likewise the 5 is taken from the factors of 30. The product of the prime factors 2, 3, 11, 7, and 5 will be the L. C. M. of the given numbers.

1. Find the L. C. M. of 12, 30, and 70.

SOLUTION.

2)	12	30	70
3)	6	15	35
5)	2	5	35
	2	1	7

Dividing by 2, it is found to be a factor of each of the numbers. It is therefore one of the factors of the L. C. M. Likewise, 3 is found to be a factor of some of the numbers, and must, therefore, be a factor of the L. C. M. Likewise, 5 is found to be a factor of some of the numbers, and is, therefore, a factor of L. C. M. The last quotients, being the remaining prime factors of the numbers, must be factors of the L. C. M. Therefore, the L. C. M. must be $2\times3\times5\times2\times7=420$

Find by either method the L. C. M.—

2. Of 4, 6, 8, and 10?
3. Of 3, 5, 9, and 10?
4. Of 5, 7, 9, and 12?
5. Of 2, 4, 6, 9, and 18?
6. Of 8, 12, 15, and 20?
7. Of 2, 3, 4, 5, and 6?
8. Of 5, 6, 7, 8, and 9?
9. Of 6, 7, 8, 9, and 10?
10. Of 7, 8, 9, 10, and 11?
11. Of 8, 9, 10, 11, and 12?
12. Of 18, 24, 30, and 36?
13. Of 9, 11, 14, and 16?
14. Of 3, 5, 7, 11, and 13?
15. Of 28, 42, and 70?
16. Of 25, 50, 75, 125, and 150?
17. Of 42, 49, and 56?
18. Of 4, 8, 16, 32, and 64?
19. Of 54, 81, 108, and 135?
20. Of 39, 51, 52, and 68?
21. Of 60, 76, 114, and 120?

22. Write two rules for finding the least common multiple of two or more numbers.

From these solutions may be derived the following—

PRINCIPLES.

1. A multiple of a number contains each of the prime factors of that number.

2. A common multiple of two or more numbers contains each of the prime factors of those numbers.

3. The L. C. M. contains each of the prime factors of those numbers, and no other factors.

4. The L. C. M. of numbers prime to each other is their product.

A FRACTION is one or more of the equal parts of a unit or quantity.

THE UNIT OF A FRACTION is the unit or quantity that is divided into equal parts.

A FRACTIONAL UNIT is one of the equal parts into which the unit or quantity is divided.

THE TERMS OF A FRACTION are its numerator and its denominator.

THE DENOMINATOR of a fraction shows into how many equal parts the unit is divided.

THE NUMERATOR of a fraction shows how many of these equal parts are taken.

A fraction indicates division; the numerator being the dividend, and the denominator being the divisor.

THE VALUE OF A FRACTION is the quotient of the numerator divided by the denominator.

Fractions are divided into two classes—COMMON and DECIMAL.

A COMMON FRACTION is expressed by writing the numerator above the denominator, with a line between.

Common fractions consist of three principal classes—SIMPLE, COMPOUND, and COMPLEX.

A SIMPLE FRACTION is one whose terms are whole numbers.

A PROPER FRACTION is a simple fraction whose numerator is less than its denominator.

An IMPROPER FRACTION is a simple fraction whose numerator equals or exceeds its denominator.

A COMPOUND FRACTION is a fraction of a fraction.

A COMPLEX FRACTION is one having a fraction in one or both of its terms.

A MIXED NUMBER is composed of a whole number and a fraction.

THE RECIPROCAL OF A NUMBER is 1 divided by that number.

THE RECIPROCAL OF A FRACTION is 1 divided by the fraction, or the fraction inverted.

1. Reduce $5\frac{3}{4}$ to an improper fraction.

SOLUTION.

In 1 are $\frac{4}{4}$; in 5 are 5 times $\frac{4}{4}$, or $\frac{20}{4}$; and $\frac{3}{4}$ added to $\frac{20}{4}$ gives $\frac{23}{4}$. It is readily observed that this result is found—

1. By multiplying the whole number by the denominator.
2. By adding the numerator to the product.
3. By writing the sum as a numerator over the denominator.

Reduce to improper fractions—

2. $4\frac{3}{5}$	10. $16\frac{2}{3}$	18. $24\frac{3}{7}$
3. $5\frac{6}{8}$	11. $24\frac{5}{12}$	19. $253\frac{5}{9}$
4. $6\frac{1}{7}$	12. $36\frac{5}{9}$	20. $4\frac{9}{10}$
5. $7\frac{8}{9}$	13. $19\frac{5}{7}$	21. 4.9
6. $8\frac{1}{3}$	14. $4\frac{5}{9}$	22. 7.05
7. $7\frac{1}{2}$	15. $8\frac{4}{7}$	23. 12.5
8. $9\frac{3}{8}$	16. $12\frac{5}{6}$	24. 1.75
9. $12\frac{4}{5}$	17. $5\frac{9}{10}$	25. 3.25

26. Reduce $\frac{625}{7}$ to a whole or to a mixed number.

SOLUTION.

Since 7 sevenths=1, 625 sevenths must equal as many ones as 7 sevenths are contained times in 625 sevenths, or as 7 is contained times in 625, which are $89\frac{2}{7}$ times. Hence $\frac{625}{7} = 89\frac{2}{7}$. Therefore, to reduce an improper fraction to a whole or to a mixed number, divide its numerator by its denominator.

Reduce the following to whole or to mixed numbers—

27. $\frac{96}{12}$	32. $\frac{524}{10}$	37. $\frac{659}{47}$
28. $\frac{124}{9}$	33. $\frac{926}{30}$	38. $\frac{8007}{100}$
29. $\frac{836}{5}$	34. $\frac{445}{17}$	39. $\frac{472}{3}$
30. $\frac{729}{11}$	35. $\frac{8324}{1000}$	40. $\frac{624}{8}$
31. $\frac{486}{13}$	36. $\frac{708}{10}$	41. $\frac{5207}{100}$

42. How may an integer be expressed in fractional form?

43. Write the rule for changing a mixed number to an improper fraction.

44. Write the rule for changing an improper fraction to a whole or mixed number.

1. Reduce $\frac{492}{744}$ to its lowest terms.

SOLUTION.

$\frac{492}{744}=\frac{12\times41}{12\times62}=\frac{41}{62}$. By factoring both terms and canceling the common factors, the fraction is reduced to its lowest terms. This process is the same as dividing both terms by their G. C. D. The same result is reached by dividing by any common divisors until the terms are prime to each other.

Reduce to lowest terms—

2. $\frac{4}{8}$ **8.** $\frac{72}{144}$ **14.** $\frac{35}{105}$ **20.** $\frac{364}{420}$

3. $\frac{3}{9}$ **9.** $\frac{12}{72}$ **15.** $\frac{144}{630}$ **21.** $\frac{374}{561}$

4. $\frac{3}{12}$ **10.** $\frac{91}{143}$ **16.** $\frac{816}{360}$ **22.** $\frac{728}{819}$

5. $\frac{24}{28}$ **11.** $\frac{49}{343}$ **17.** $\frac{323}{391}$ **23.** $\frac{616}{1120}$

6. $\frac{32}{48}$ **12.** $\frac{54}{88}$ **18.** $\frac{308}{506}$ **24.** $\frac{924}{1618}$

7. $\frac{60}{90}$ **13.** $\frac{316}{395}$ **19.** $\frac{299}{713}$ **25.** $\frac{756}{1140}$

26. Reduce $\frac{5}{8}$ to 24ths.

SOLUTION.

$\frac{5\times3}{8\times3}=\frac{15}{24}$ The required denominator is 3 times greater than the given denominator. The numerator also must be 3 times greater than the given numerator. Multiplying both terms of the fraction by the same number does not change the value of the fraction.

27. Reduce $\frac{4}{5}$ to 20ths. **31.** Reduce $\frac{3}{4}$ and $\frac{2}{3}$ to 12ths.

28. Reduce $\frac{6}{7}$ to 28ths. **32.** Reduce $\frac{4}{7}$ and $\frac{5}{8}$ to 56ths.

29. Reduce $\frac{5}{6}$ to 30ths. **33.** Reduce $\frac{5}{6}$ and $\frac{3}{4}$ to 36ths.

30. Reduce $\frac{7}{12}$ to 60ths. **34.** Reduce $\frac{3}{7}$ and $\frac{5}{9}$ to 63rds.

35. Reduce $\frac{3}{4}$, $\frac{4}{5}$, $1\frac{1}{12}$, $1\frac{3}{15}$, and $\frac{19}{20}$ to a common denominator.

SOLUTION.

The L. C. M. of the denominators 4, 5, 12, 15, and 20 is 60. Taking this as the common denominator, $\frac{3}{4}=\frac{45}{60}$; $\frac{4}{5}=\frac{48}{60}$; $1\frac{1}{12}=\frac{65}{60}$; $1\frac{3}{15}=\frac{62}{60}$; $\frac{19}{20}=\frac{57}{60}$.

36. Reduce $\frac{8}{5}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{6}{20}$, and $1\frac{1}{30}$ to a common denominator.

37. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{2}{12}$ and $\frac{9}{14}$ to a common denominator.

1. Change the fraction $\frac{3}{4}$ to a decimal.

SOLUTION.

The decimal must have a denominator of 10, 100, or other power of 10. Hence the given denominator 4 must be multiplied by some integer to make the required denominator. Since 4 is not an exact divisor of 10, it must be multiplied by a number to change it to 100. The required multiplier is seen to be 25. The numerator must also be multiplied by 25, which changes the fraction to $\frac{75}{100}$, expressed by the decimal .75.

Since annexing ciphers to both terms of a fraction does not change its value, the $\frac{3}{4}$ may be written $\frac{300}{400}$. This may now be reduced to *hundredths* by dividing by 4, which gives $\frac{75}{100}$, or .75.

4)3.00 This is seen to be the same as annexing ciphers to the
.75 numerator and dividing by the denominator.

Reduce to decimals—

2.	$\frac{4}{5}$	**7.**	$\frac{9}{16}$	**12.**	$\frac{7}{8}$
3.	$\frac{6}{8}$	**8.**	$\frac{11}{32}$	**13.**	$\frac{3}{800}$
4.	$\frac{3}{12}$	**9.**	$\frac{15}{16}$	**14.**	$\frac{3}{64}$
5.	$\frac{4}{25}$	**10.**	$\frac{3}{40}$	**15.**	$\frac{43}{50}$
6.	$\frac{3}{192}$	**11.**	$\frac{3}{1000}$	**16.**	$\frac{7}{16}$

17. Reduce .125 to a common fraction.

SOLUTION.

.125=$\frac{125}{1000}$, which reduced to lowest terms is $\frac{1}{8}$.

18.	.16	**22.**	.325	**26.**	.0225
19.	.25	**23.**	.675	**27.**	.0925
20.	.625	**24.**	.0125	**28.**	.0425
21.	.125	**25.**	.105	**29.**	.33$\frac{1}{3}$

30. Write the rule for reducing common fractions to the lowest terms.

31. Write the rule for changing a common fraction to a decimal.

32. Write the rule for changing a decimal to a common fraction.

33. Write the rule for reducing common fractions to a common denominator.

ADDITION AND SUBTRACTION OF FRACTIONS.

In addition of whole numbers, it was seen that units can be added only to units, tens to tens, etc. Thus, 3 units and 4 tens will make neither 7 units nor 7 tens; but 4 tens equal 40 units; and 40 units+3 units=43 units.

So in fractions, 3 *fourths* cannot be added to 4 *sixths*, for the result will be neither 7 *fourths* nor 7 *sixths*. But 3 fourths equal 9 *twelfths* and 4 sixths equal 8 *twelfths;* and 9 twelfths and 8 twelfths=17 twelfths, or 1 and 5 twelfths; that is, $\frac{3}{4}+\frac{4}{6}=\frac{9}{12}+\frac{8}{12}=1\frac{7}{12}=1\frac{5}{12}$. Hence, for the addition of fractions, *reduce the given fractions to a common denominator. Over this denominator place the sum of their numerators.*

Likewise, for the subtraction of fractions, reduce the given fractions to a common denominator and over this denominator place the difference of their numerators.

Find the *least* common denominator of the fractions. If the result be an improper fraction, it must be reduced to a whole or mixed number.

Find both the sum and difference of—

1.	$\frac{1}{2}$ and $\frac{1}{3}$	**14.**	$\frac{5}{6}$ and $\frac{7}{8}$	**27.**	$1\frac{1}{2}$ and $2\frac{1}{3}$
2.	$\frac{1}{3}$ and $\frac{1}{4}$	**15.**	$\frac{7}{8}$ and $1\frac{1}{2}$	**28.**	$1\frac{1}{2}$ and $3\frac{1}{4}$
3.	$\frac{1}{2}$ and $\frac{1}{6}$	**16.**	$\frac{5}{6}$ and $1\frac{1}{2}$	**29.**	$2\frac{1}{3}$ and $3\frac{1}{4}$
4.	$\frac{1}{3}$ and $\frac{1}{6}$	**17.**	$\frac{2}{3}$ and $1\frac{1}{2}$	**30.**	$4\frac{1}{3}$ and $1\frac{1}{2}$
5.	$\frac{1}{6}$ and $\frac{1}{8}$	**18.**	$\frac{2}{3}$ and $\frac{5}{6}$	**31.**	$4\frac{1}{3}$ and $2\frac{1}{2}$
6.	$\frac{1}{8}$ and $\frac{1}{10}$	**19.**	$\frac{2}{3}$ and $\frac{3}{4}$	**32.**	$4\frac{1}{3}$ and $3\frac{1}{4}$
7.	$\frac{2}{3}$ and $\frac{3}{4}$	**20.**	$\frac{2}{3}$ and $\frac{5}{8}$	**33.**	$1\frac{1}{3}$ and $2\frac{1}{2}$
8.	$\frac{2}{3}$ and $\frac{3}{5}$	**21.**	$\frac{2}{5}$ and $\frac{3}{10}$	**34.**	$1\frac{1}{3}$ and $3\frac{1}{3}$
9.	$\frac{3}{4}$ and $\frac{3}{5}$	**22.**	$\frac{3}{5}$ and $\frac{2}{3}$	**35.**	$1\frac{1}{3}$ and $4\frac{1}{4}$
10.	$\frac{1}{6}$ and $\frac{1}{6}$	**23.**	$\frac{2}{3}$ and $\frac{3}{4}$	**36.**	$3\frac{1}{3}$ and $6\frac{1}{4}$
11.	$\frac{1}{8}$ and $\frac{1}{8}$	**24.**	$\frac{2}{3}$ and $\frac{2}{7}$	**37.**	$5\frac{1}{8}$ and $6\frac{1}{8}$
12.	$\frac{2}{3}$ and $\frac{2}{8}$	**25.**	$\frac{2}{7}$ and $\frac{5}{6}$	**38.**	$6\frac{1}{8}$ and $7\frac{1}{9}$
13.	$\frac{2}{5}$ and $\frac{3}{8}$	**26.**	$\frac{3}{4}$ and $\frac{4}{7}$	**39.**	$9\frac{2}{3}$ and $10\frac{3}{4}$

40. Write the rule for the addition of common fractions.

41. Write the rule for the subtraction of common fractions.

1. If 1 arithmetic costs $\frac{3}{10}$ of a dollar, what will 10 cost?

SOLUTION.

The cost of 10 books is 10 times the cost of 1 book; and 10 times 3 *tenths* of a dollar are 30 *tenths* of a dollar, expressed in fractional form, $\frac{30}{10}$, which reduced to a whole number, equals 3 dollars. That is, to multiply a fraction, multiply the numerator, and then reduce to lowest terms, or to a whole or mixed number. The work may be shortened by cancellation, thus:

$$\$\frac{3}{\cancel{10}} \times \frac{\cancel{10}}{1} = \$3.$$

The same process is followed when either one or both factors are fractions.

2. Multiply $\frac{2}{3}$ by $\frac{9}{20}$.

SOLUTION.

$\frac{2}{3} \times \frac{9}{20} = \frac{18}{60}$
$\frac{18}{60} = \frac{3}{10}$
3
$\frac{\cancel{2} \times \cancel{9}}{\cancel{3} \times \cancel{20}} = \frac{3}{10}$
10

Multiply the numerators for a new numerator, and the denominators for a new denominator, and reduce to lowest terms. The same result is reached by cancellation.

Find the product—

3. Of $\frac{1}{3}$ and $\frac{2}{5}$ **10.** Of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ **17.** Of $\frac{2}{3}$ of $\frac{4}{5}$*and $\frac{7}{10}$ of $\frac{5}{8}$

4. Of $\frac{1}{3}$ and $\frac{5}{8}$ **11.** Of $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ **18.** Of $\frac{4}{7}$ of $1\frac{1}{2}$ and $\frac{2}{3}$ of $1\frac{1}{2}$

5. Of $\frac{4}{5}$ and $\frac{10}{16}$ **12.** Of $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ **19.** Of $\frac{3}{4}$ of $\frac{5}{6}$ and $\frac{8}{9}$ of $\frac{3}{10}$

6. Of $\frac{4}{9}$ and $\frac{18}{27}$ **13.** Of $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ **20.** Of $3\frac{1}{2}$ of $4\frac{1}{3}$ and $5\frac{1}{4}$

7. Of $\frac{8}{11}$ and $\frac{55}{56}$ **14.** Of $\frac{3}{7}$, $\frac{8}{9}$ and $\frac{7}{10}$ **21.** Of $\frac{1}{8}$ of $3\frac{1}{3}$ and $4\frac{1}{8}$

8. Of $\frac{7}{9}$ and $\frac{63}{77}$ **15.** Of $\frac{5}{6}$, $\frac{4}{7}$ and $\frac{6}{15}$ **22.** Of $3\frac{1}{3}$ of $4\frac{1}{4}$ and $5\frac{1}{7}$

9. Of $\frac{13}{14}$ and $\frac{28}{42}$ **16.** Of $\frac{2}{4}$, $\frac{7}{9}$ and $\frac{16}{21}$ **23.** Of $\frac{1}{2}$ of $4\frac{1}{2}$ and $7\frac{1}{2}$

24. Find the cost of $5\frac{1}{2}$ yards of cloth at $\$2\frac{1}{2}$ per yard.

25. Find the cost of $2\frac{1}{2}$ pounds of coffee at $\$\frac{2}{3}$ per pound.

26. Find the cost of $10\frac{1}{2}$ pounds of butter at $\$\frac{1}{3}$ per pound.

27. Find the cost of $5\frac{1}{2}$ acres of land at $\$12\frac{1}{2}$ per acre.

28. Find the cost of $2\frac{7}{10}$ tons of hay at $\$16\frac{2}{3}$ per ton.

* A compound fraction is an indicated multiplication.

1. Divide $\frac{4}{7}$ by $\frac{5}{8}$.

SOLUTION I.

If the fractions be reduced to a common denominator, their numerators may be operated upon as if they were whole numbers. Thus, $\frac{4}{7}=\frac{32}{56}$; $\frac{5}{8}=\frac{35}{56}$. And $\frac{32}{56}$ divided by $\frac{35}{56}$ is the same as 32 divided by 35, or $\frac{32}{35}$.

SOLUTION II.

$\frac{4}{7}$ divided by 1 will give $\frac{4}{7}$ for a quotient; divided by $\frac{1}{8}$ will give 8 times as large a quotient as when divided by 1, or $\frac{4}{7}\times\frac{8}{1}=\frac{32}{7}$; divided by $\frac{5}{8}$, will give one fifth as large a quotient as when divided by $\frac{1}{8}$, or $\frac{1}{5}\times\frac{32}{7}=\frac{32}{35}$, the same result as found by the first method. In fact, the dividend $\frac{4}{7}$ has been multiplied by $\frac{8}{5}$; that is, by the divisor with its terms inverted. Hence, for dividing one fraction by another,

Invert the terms of the divisor, and proceed as in multiplication.

If either dividend or divisor be a whole number, make it an improper fraction by writing 1 for a denominator.

Find the quotient—

2. Of $\frac{1}{2}\div\frac{1}{4}$ **7.** Of $\frac{6}{7}\div\frac{6}{1}$ **12.** Of $3\frac{1}{3}\div 10$

3. Of $\frac{1}{5}\div\frac{1}{6}$ **8.** Of $4\frac{1}{2}\div 2\frac{1}{4}$ **13.** Of $\frac{1}{2}$ of $\frac{3}{4}\div\frac{1}{6}$ of $\frac{5}{8}$

4. Of $\frac{1}{4}\div\frac{1}{2}$ **9.** Of $17\frac{1}{3}\div 4\frac{1}{3}$ **14.** Of $2\frac{1}{2}$ of $3\frac{1}{2}\div 4\frac{1}{3}$

5. Of $\frac{1}{6}\div\frac{1}{5}$ **10.** Of $3\frac{1}{3}\div 2\frac{1}{2}$ **15.** Of $2\frac{1}{2}$ of $\frac{8}{8}\div\frac{1}{8}$ of $2\frac{1}{2}$

6. Of $\frac{2}{3}\div\frac{4}{5}$ **11.** Of $10\div 3\frac{1}{3}$ **16.** Of $3\frac{1}{5}$ of $\frac{3}{4}\div\frac{1}{7}$ of $4\frac{1}{5}$

Complex fractions may be regarded as indicating that the numerator is to be divided by the denominator.

17. Reduce $\frac{3\frac{3}{5}}{5\frac{1}{7}}$ to a simple fraction.

SOLUTION.

$$\frac{3\frac{3}{5}}{5\frac{1}{7}}=3\tfrac{3}{5}\div 5\tfrac{1}{7}=\frac{18\times 7}{5\times 36}=\frac{7}{10}.$$

18. $\dfrac{6\frac{2}{3}}{7\frac{1}{2}}$ **20.** $\dfrac{3.5}{.2\overline{8}}$ **22.** $\dfrac{41\frac{2}{3}}{50}$

19. $\dfrac{25\frac{4}{9}}{31\frac{1}{6}}$ **21.** $\dfrac{6.3}{2.7}$ **23.** $\dfrac{50}{58\frac{1}{3}}$

ALIQUOT PARTS OF A DOLLAR.

cts.	$	cts.	$	cts.	$	cts.	$
5	$=\frac{1}{20}$	$16\frac{2}{3}$	$=\frac{1}{6}$	$37\frac{1}{2}$	$=\frac{3}{8}$	$66\frac{2}{3}$	$=\frac{2}{3}$
$6\frac{1}{4}$	$=\frac{1}{16}$	20	$=\frac{1}{5}$	50	$=\frac{1}{2}$	75	$=\frac{3}{4}$
$8\frac{1}{3}$	$=\frac{1}{12}$	25	$=\frac{1}{4}$	$56\frac{1}{4}$	$=\frac{9}{16}$	$83\frac{1}{3}$	$=\frac{5}{6}$
10	$=\frac{1}{10}$	$31\frac{1}{4}$	$=\frac{5}{16}$	$58\frac{1}{3}$	$=\frac{7}{12}$	$87\frac{1}{2}$	$=\frac{7}{8}$
$12\frac{1}{2}$	$=\frac{1}{8}$	$33\frac{1}{3}$	$=\frac{1}{3}$	$62\frac{1}{2}$	$=\frac{5}{8}$		

1. Find the cost of 1200 boxes of fruit at $12\frac{1}{2}$ cents a box.
2. Find the cost of 1200 boxes of fruit at $16\frac{2}{3}$ cents a box.
3. Find the cost of 1200 boxes of fruit at 20 cents a box.
4. Find the cost of 1200 boxes of fruit at 25 cents a box.
5. Find the cost of 1200 boxes of fruit at $8\frac{1}{3}$ cents a box.
6. Find the cost of 1200 boxes of fruit at $33\frac{1}{3}$ cents a box.
7. Find the cost of 1200 boxes of fruit at 50 cents a box.
8. Find the cost of 1200 boxes of fruit at 75 cents a box.
9. Find the cost of 1200 boxes of fruit at $37\frac{1}{2}$ cents a box.
10. Find the cost of 1200 boxes of fruit at $62\frac{1}{2}$ cents a box.
11. Find the cost of 1200 boxes of fruit at $83\frac{1}{3}$ cents a box.
12. Find the cost of 1200 boxes of fruit at $87\frac{1}{2}$ cents a box.
13. How many boxes of fruit will $35 buy at $6\frac{1}{4}$ cents a box?
14. How many boxes of fruit will $35 buy at $12\frac{1}{2}$ cents a box?
15. How many boxes of fruit will $35 buy at $8\frac{1}{3}$ cents a box?
16. How many boxes of fruit will $35 buy at $16\frac{2}{3}$ cents a box?
17. How many boxes of fruit will $35 buy at 20 cents a box?
18. How many boxes of fruit will $35 buy at 25 cents a box?
19. What will 1200 yards of goods cost at $1.20 a yard?

SOLUTION.

At $1.00 a yard, 1200 yards cost .....
At $\frac{1}{5}$ a yard, 1200 yards cost .....
At $1.20 a yard, 1200 yards cost .....

20. What will 1200 yards of goods cost at $1.05 a yard?
21. What will 1200 yards of goods cost at $1.10 a yard?
22. What will 1200 yards of goods cost at $1.12\frac{1}{2}$ a yard?
23. What will 1200 yards of goods cost at $1.06\frac{1}{4}$ a yard?

1. Reduce $\frac{4\frac{7}{8}\times 3\frac{1}{4}\times 6\frac{4}{5}}{5\frac{1}{3}\times 9\frac{2}{7}}$ to a simple fraction.

2. Reduce $\frac{15\frac{3}{8}\times 7\frac{6}{17}\times 9}{7\frac{4}{17}\times 24\times 1\frac{1}{13}\frac{2}{3}}$ to a simple fraction.

3. Reduce $\frac{12\times\frac{3}{5}\times 9\frac{2}{15}}{7\frac{1}{2}\times 8\frac{4}{5}\times 6}$ to a simple fraction.

4. Reduce $\frac{14\frac{2}{7}\times 18\frac{4}{5}\times 6\frac{4}{5}}{7\frac{1}{5}\times 100\times 16}$ to a simple fraction.

5. Reduce $\frac{.025\times .75\times 4.8}{.001\times 62.5\times .9}$ to a simple fraction.

6. Reduce $\frac{.009\times .08\times 600}{48\times .06\times .54\times .036}$ to a simple fraction.

7. Reduce $\frac{.03\times .7\times .008\times .0005}{3\times 40\times .25\times 4.9}$ to a simple fraction.

8. Reduce $\frac{.0125\times 6.4\times 7000\times .0081}{.00096\times 500\times 5.6\times .016}$ to a simple fraction.

9. Reduce $\frac{\frac{1}{3}\times .25\times 2\frac{1}{2}\times .16\frac{2}{3}\times\frac{2}{5}}{\frac{1}{2}\times 2.5\times\frac{1}{6}\times\frac{3}{4}\times .8}$ to a simple fraction.

10. Reduce $\frac{37.5\times\frac{2}{3}\times .87\frac{1}{2}\times 12.5}{\frac{7}{8}\times\frac{1}{5}\times 66\frac{1}{4}\times\frac{1}{4}}$ to a simple fraction.

11. Reduce $\frac{\frac{1}{5}\times 7.5\times .625\times 20\%\text{ of }.16\frac{2}{3}}{\frac{1}{8}\times .4\times\frac{3}{4}\times\frac{1}{6}\text{ of }.2}$ to a simple fraction.

12. Reduce $\frac{125\times 90\times 3\frac{1}{3}\%\text{ of }500}{37.5\times 300\times 20\times\text{ of }9}$ to a simple fraction.

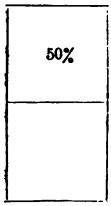
13. Reduce $\frac{.032\times\frac{1}{4}\times 56\times 720}{50\times .8\times 62.5\times 2.8\times 900}$ to a simple fraction.

14. Reduce $\frac{58\frac{1}{3}\times 600\times 62.5\times 750}{125\times 8\frac{1}{3}\times 38\frac{1}{2}\times .05}$ to a simple fraction.

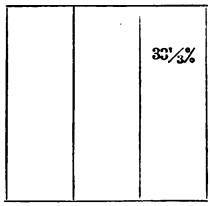
15. Reduce $\frac{16\times .04\times 5.5\times 720}{.11\times .2\times .9\times 1200}$ to a simple fraction.

16. Reduce $\frac{30\times 41\frac{2}{3}\times 4\frac{1}{2}\times 3.9}{.13\times 5\times 8\frac{1}{3}\times 30}$ to a simple fraction.

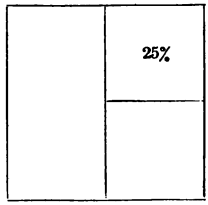
17. Reduce $\frac{9.8\times\frac{1}{6}\times 10\frac{1}{5}\times 165}{.33\frac{1}{3}\times 14\times .07\times 510}$ to a simple fraction.



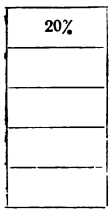
$\frac{1}{2}=50\%$



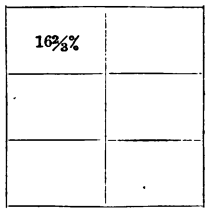
$\frac{1}{3}=33\frac{1}{3}\%$



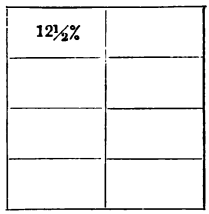
$\frac{1}{4}=25\%$



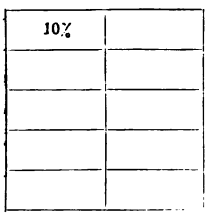
$\frac{1}{5}=20\%$



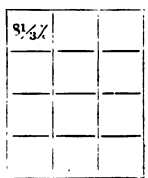
$\frac{1}{6}=16\frac{2}{3}\%$



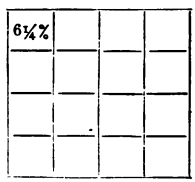
$\frac{1}{8}=12\frac{1}{2}\%$



$\frac{1}{10}=10\%$



$\frac{1}{12}=8\frac{1}{3}\%$



$\frac{1}{16}=6\frac{1}{4}\%$

1. What part of $\frac{1}{2}$ is $16\frac{2}{3}\%$? Is $12\frac{1}{2}\%$?
2. What part of $\frac{1}{4}$ is $12\frac{1}{2}\%$? Is $6\frac{1}{4}\%$?
3. What part of $\frac{1}{8}$ is $16\frac{2}{3}\%$? Is $8\frac{1}{3}\%$?
4. What part of $\frac{1}{2}$ is 10 %? Is $6\frac{1}{4}\%$?
5. How many times $8\frac{1}{3}\%$ equals 4 times $12\frac{1}{2}\%$?
6. How many times $12\frac{1}{2}\%$ equals 3 times $16\frac{1}{2}\%$?
7. How many times $8\frac{1}{3}\%$ equals 4 times $6\frac{1}{4}\%$?
8. How many times $6\frac{1}{4}\%$ equals 9 times $8\frac{1}{3}\%$?
9. How many times $16\frac{2}{3}\%$ equals 8 times $8\frac{1}{3}\%$?

When comparisons are based upon 100, the operation is called *percentage*, and one number is called a certain *per cent* of another.

PER CENT means *by the hundred.* The whole of a number is called 100 per cent of that number. And just as $\frac{1}{2}$ of a dollar is 50 cents, so $\frac{1}{2}$ of any number is 50 per cent of that number.

Per cent is indicated by the sign, %. This is but another method of expressing *hundredths*, which may be expressed by a *decimal*, a *common fraction*, or by the term *per cent*, indicated by the sign, %. Pupils will understand the application of percentage when they note that their standing in their class is indicated by 80%, 90%, etc., when a perfect record is indicated by 100.

TABLE.

50 % of a dollar = $\frac{1}{2}$ = \$.50.
$33\frac{1}{3}$% of a dollar = $\frac{1}{3}$ = \$.$33\frac{1}{3}$.
$66\frac{2}{3}$% of a dollar = $\frac{2}{3}$ = \$.$66\frac{2}{3}$.
25 % of a dollar = $\frac{1}{4}$ = \$.25.
75 % of a dollar = $\frac{3}{4}$ = \$.75.
20 % of a dollar = $\frac{1}{5}$ = \$.20.
40 % of a dollar = $\frac{2}{5}$ = \$.40.
60 % of a dollar = $\frac{3}{5}$ = \$.60.
80 % of a dollar = $\frac{4}{5}$ = \$.80.
$16\frac{2}{3}$% of a dollar = $\frac{1}{6}$ = \$.$16\frac{2}{3}$.
$12\frac{1}{2}$% of a dollar = $\frac{1}{8}$ = \$.$12\frac{1}{2}$.
$37\frac{1}{2}$% of a dollar = $\frac{3}{8}$ = \$.$37\frac{1}{2}$.
$62\frac{1}{2}$% of a dollar = $\frac{5}{8}$ = \$.$62\frac{1}{2}$.
$87\frac{1}{2}$% of a dollar = $\frac{7}{8}$ = \$.$87\frac{1}{2}$.
$8\frac{1}{3}$% of a dollar = $\frac{1}{12}$ = \$.$08\frac{1}{3}$.
$41\frac{2}{3}$% of a dollar = $\frac{5}{12}$ = \$.$41\frac{2}{3}$.
$6\frac{1}{4}$% of a dollar = $\frac{1}{16}$ = \$.$06\frac{1}{4}$.
5 % of a dollar = $\frac{1}{20}$ = \$.05.

Find the cost—

1. Of 50% of 48 yards of goods at $12\frac{1}{2}$ cents, per yard.

SOLUTION.

50% of 48 yards = $\frac{1}{2}$ of 48 yards, or 24 yards.
1 yard costs $12\frac{1}{2}$ cents, or \$$\frac{1}{8}$.
24 yards cost 24 times \$$\frac{1}{8}$ = \$$\frac{24}{8}$, or \$3.00.

2. Of 50 % of 150 acres of land at \$50 per acre.
3. Of 40 % of 1600 bushels of grain at $33\frac{1}{3}$¢ per bushel.
4. Of $33\frac{1}{3}$% of 1440 bushels of grain at $33\frac{1}{3}$¢ per bushel.
5. Of 25 % of 5600 pounds of grain at \$1.25 per cental.
6. Of 20 % of 2400 pounds of coffee at $37\frac{1}{2}$¢ per pound.
7. Of $16\frac{2}{3}$% of 4200 pounds of coffee at $37\frac{1}{2}$¢ per pound.

PERCENTAGE, as a process, treats of computing in hundredths.

The *rate* is the number of hundredths.

The *base* is the number of which the hundredths are taken.

The *percentage*, as a quantity, is the product of the base multiplied by the rate.

1. How many acres are 5% of 400 acres?

SOLUTION.

400 acres multiplied by .05, equal 20 acres, the percentage.

The *base* multiplied by the *rate* gives the *percentage*.

2. What per cent of 400 acres are 20 acres?

SOLUTION.

Stated in the order of the previous solution, 400 acres multiplied by — will give 20 acres. The missing factor is found by dividing the product 20 by the given factor 400, which gives .05, or 5% for the rate.

The *percentage* divided by the *base* gives the *rate*.

It is also evident that the *percentage* divided by the *rate* gives the *base*.

3. What % of 75 is 25?

4. What % of .15 is .05?

5. What % of 400 is 100?

6. What % of 400 is 50?

7. What % of 400 is 500?

8. What % of 400 is 800?

9. What % of 160 is 20?

10. What % of 400 is 4?

11. What % of 400 is .4?

12. What % of 250 is $6\frac{1}{2}$?

13. What % of 250 is $6\frac{1}{4}$?

14. What % of 1100 is 88?

15. What % of 1180 is 59?

16. What % of 2500 is 150?

17. What % of 5000 is 600?

18. 200 is $37\frac{1}{2}$% of——

19. 450 is 5 % of——

20. 160 is $33\frac{1}{3}$% of——

21. 125 is $16\frac{2}{3}$% of——

22. 1200 is $33\frac{1}{3}$% of——

23. 1200 is 200 % of——

24. 1200 is $133\frac{1}{3}$% of——

25. 1200 is 150 % of——

26. 1.25 is 20 % of——

27. 12.5 is 25 % of——

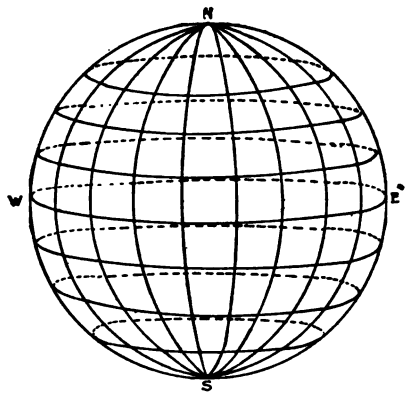
28. 12.5 is $12\frac{1}{2}$% of——

29. 963 is 9 % of——

30. 6220 is 200 % of——

31. 1525 is 25 % of——

32. 1525 is $2\frac{1}{2}$ % of——

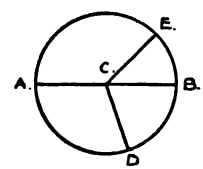


A POINT is position without length, breadth or thickness.

A LINE is the path of a moving point.

If the path continues in the same direction, it is a *straight* line; if it changes direction, it is a *curved* line.

A CIRCLE is a figure bounded by a curved line called the *circumference*, all points of which are equally distant from a point within called the center.



AN ARC is any portion of the circumference, as, A E or B D. An arc equal to a quarter of the circumference is called a *quadrant.*

A RADIUS is a line drawn from the center to the circumference; as C A or C B.

A DIAMETER is a straight line drawn *through* the center and limited by the circumference; as A B.

The circumference of a a circle is divided into 360 equal parts, called *degrees*, and these are subdivided as shown in the table.

LATITUDE is distance north or south of the equator.

LONGITUDE is the distance east or west of a given meridian, usually reckoned from the meridian passing through Greenwich, England.

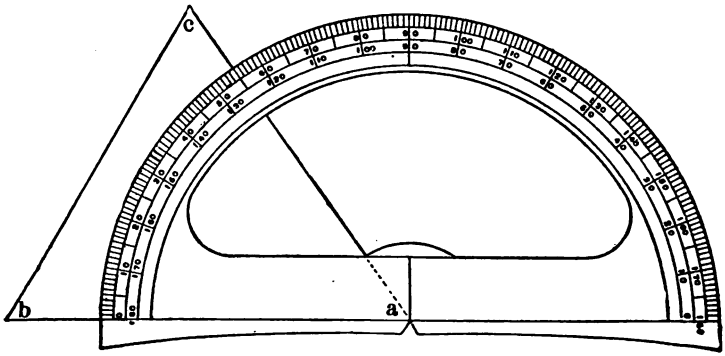


TABLE FOR MEASURING CIRCLES.

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
30 degrees = 1 sign.
12 signs = 1 circle.
360 degrees = 1 circumference (C.)

A *degree of the earth's equator* contains 60 geographical miles, or about 69⅛ common miles. As a matter of convenience, seamen estimate distances by the latitude and longitude passed over. Thus, if a vessel sails over *1 minute* of longitude on the equator, it is reckoned as *1 mile;* a *degree* is reckoned as *60 miles.* The actual distance in common miles is found by multiplying 69.16 by the number of degrees.

A *right angle* is an angle of 90°.

A *sextant* is an arc of 60°, or ⅙ of a circumference.

A *quadrant* is an arc of 90°, or ¼ of a circumference.

A *semi-circumference* is an arc of 180°, or ½ of a circumference.

1. What per cent of a circumference is ⅛ of a quadrant?
2. What per cent of a sextant is ½ of a quadrant?
3. What per cent of a quadrant is ⅓ of a sign?
4. What per cent of a sign is ¼ of a sextant?
5. What per cent of a quadrant is ½ of a sign?

Refer to a globe or to a geography and—

1. Locate the place from which longitude is reckoned.

2. Locate the place from which latitude is reckoned.

3. Find the latitude of the North Pole. Of the South Pole.

4. Find a place having neither latitude nor longitude.

5. Find the place 90° east of your home, on the same latitude.

6. Find the place 180° east of your home, on the same latitude.

7. Find the place on the other side of the earth from your home, on a direct line through the center.

8. Find a place from which if you travel 1,500 miles north, and then 1,500 miles south, you will be 3,000 miles from your starting point.

9. Find a place from which if you go 1,500 miles north, thence 1,500 west, thence 1,500 miles south, you will be at your starting point.

10. Find a place from which if you travel 1,500 miles south, thence 1,500 miles east, thence 1,500 miles north, you will again be at your starting point.

11. Find a place from which two persons may travel 1,500 miles in the same direction, and then be 3,000 miles apart.

12. Determine the latitude and longitude of San Francisco.

13. Determine the latitude and longitude of Moscow.

14. Determine the latitude and longitude of Rio Janeiro.

15. Determine the latitude and longitude of Melbourne.

16. Find two important cities having a difference of about 60° latitude.

17. Find two important cities having a difference of about 90° longitude.

18. Find two important cities having the same latitude.

19. Find two important cities having the same longitude.

20. Find some meridian of longitude that does not pass through any large land mass.

TABLE OF LATITUDE AND LONGITUDE.

CITIES.	LATITUDE.		LONGITUDE.	
Albany	42°39′50″	N,	73°44′56″	W.
Atlanta	33°45′	N,	84°25′	W.
Berlin	52°30′	N,	13°24′	E.
Boston	42°21′	N,	71° 4′	W.
Buenos Ayres	34°36′	S,	58°22′	W.
Calcutta	22°34′49″	N,	88°27′16″	E.
Chicago	41°50′	N,	87°37′30″	W.
Cleveland	41°31′	N,	81°46′30″	W.
Denver	39°47′	N,	104°59′23″	W.
Havana, Cuba	23° 8′	N,	82°22′	W.
Honolulu	21°18′12″	N,	157°52′	W.
Manila	14°36′	N,	120°58′	E.
Melbourne	37°48′	S,	144°58′	E.
Montreal	45°31′	N,	73°35′	W.
New Orleans	29°57′	N,	90° 3′28″	W.
New York	40°42′43″	N,	74° 3′	W.
Paris	48°50′12″	N,	2°20′22″	E.
Pekin	39°54′13″	N,	116°28′54″	E.
Pretoria	25°24′	S,	28°45′	E.
Quito	13′	S,	78°43′	W.
Rio Janeiro	22°54′	S,	43°10′	W.
Rome	41°53′52″	N,	12°28′40″	E.
St. Petersburg	59°56′30″	N,	30°19′	E.
San Francisco	37°47′55″	N,	122°24′32″	W.
Seattle	47°36′	N,	122°20′	W.
Valparaiso	31° 1′56″	S,	71°41′45″	W.
Washington, D. C	38°53′39″	N,	77°00′36″	W.
Wellington, N. Z	41°17′	S,	174°47′	E.

Determine as nearly as possible the latitude and longitude of your nearest town or city.

State the latitude of the several zones of the earth's surface.

How find the difference in longitude between two places, if one is east and the other west longitude?

How find the difference in longitude between two places, if both are east, or both west longitude?

How find the difference in latitude between two places, if both are on the same side of the equator? If both are not on the same side of the equator?

Refer to the table and state the difference—

1. In latitude between Albany and New Orleans.
2. In latitude between Chicago and Havana.
3. In latitude between Valparaiso and San Francisco.
4. In longitude between Seattle and St. Petersburg.
5. In longitude between San Francisco and Honolulu.
6. In longitude between Buenos Ayres and Chicago.
7. In latitude between Rome and Melbourne.
8. In latitude between Honolulu and Calcutta.
9. In latitude between New York and Buenos Ayres.
10. In longitude between Havana and Valparaiso.
11. In longitude between Melbourne and Montreal.
12. In longitude between New York and Pretoria.
13. In latitude between Paris and Quito.
14. In latitude between St. Petersburg and Pretoria.
15. In longitude between Wellington and Paris.
16. In longitude between Albany and Berlin.
17. In longitude between Atlanta and Boston.
18. In latitude between Seattle and Manila.
19. In latitude between Washington and Montreal.
20. In latitude between Atlanta and Rio Janeiro.
21. In latitude between Berlin and Wellington.
22. In longitude between Calcutta and Manila.
23. In longitude between Denver and Rome.
24. In longitude between Cleveland and New Orleans.
25. In latitude between Denver and Pekin.
26. In latitude between Boston and Cleveland.

Since the earth turns upon its axis once in 24 hours, it follows that $\frac{1}{24}$ of 360°, or 15° of longitude pass under the sun in 1 hour, and $\frac{1}{60}$ of 15°, or 15′, pass under the sun in 1 minute, and $\frac{1}{60}$ of 15′, or 15″ pass under the sun in 1 second of time.

As the earth revolves from west to east, places east of a given meridian have mid-day sooner, and have earlier, or faster time, because the sun appears to them earlier; and, for similar reasons places west have later or slower time.

1. Express 20°36′15″ of longitude in time.

SOLUTION.

15)20°36′15″

1 hr. 22 min. 25 sec.

Since 15° longitude give 1 hour in time, 15′ longitude 1 minute, and 15″ longitude 1 second, divide 20°36′15″ by 15, as in compound division, and the quotient will be the time required.

2. Express 1 hour 4 minutes 4 seconds in degrees.

SOLUTION.

1 hr. 4 min. 4 sec.
15

16° 1′ 0″

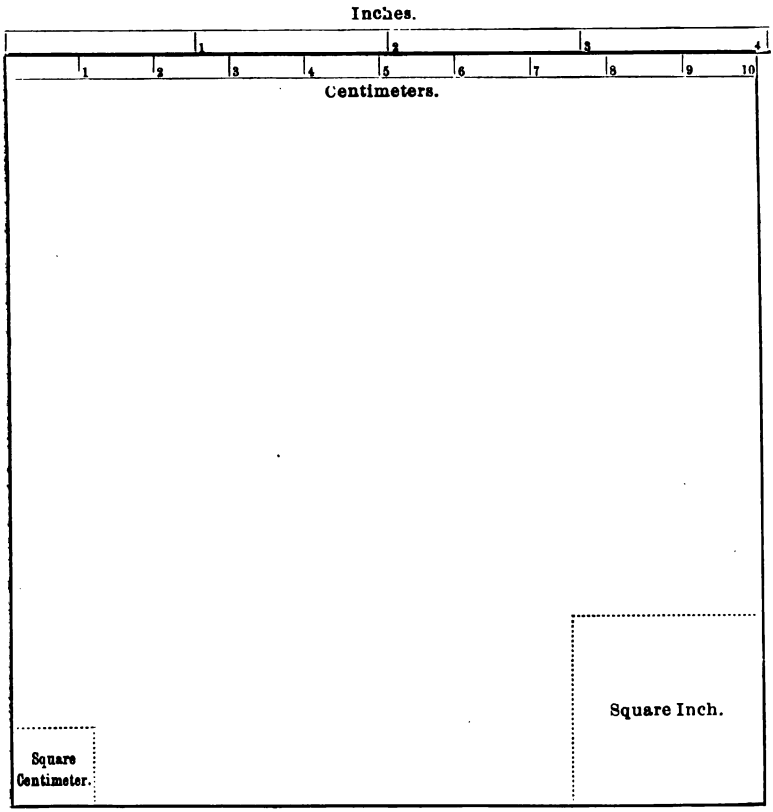
Since longitude is changed to time by division, time is reduced to longitude by multiplication. Therefore, multiply 1 hour 4 minutes 4 seconds by 15, as in compound multiplication, and the product will be the longitude required.

Hence, if longitude is expressed in *degrees, divide by 15*, and the quotient will give the longitude in *hours, minutes and seconds.*

If longitude is expressed in *time, multiply by 15;* the product will give the longitude in *degrees, minutes and seconds.*

Find the difference in time between—

3. Albany and Chicago.
4. Berlin and Paris.
5. Calcutta and Paris.
6. Boston and Cleveland.
7. Boston and Seattle.
8. Washington and Seattle.
9. Berlin and Honolulu.
10. San Francisco and Chicago.
11. Cleveland and San Francisco.
12. New York and New Orleans.
13. Washington and Valparaiso.
14. Rome and San Francisco.
15. New York and St. Petersburg.
16. Melbourne and Montreal.



The area of this figure is 1 square *decimeter*.

A cubic decimeter, in measuring capacity, is called a *liter*.

A liter of water at 4° C. weighs a *kilogram*.

A cubic centimeter of water at 4° C. weighs a *gram*.

THE METRIC SYSTEM.

IN THE METRIC SYSTEM OF WEIGHTS AND MEASURES, the principal denomination is the *Meter.*

THE METER. The fundamental unit of length in the Metric System, originally defined as one ten-millionth of the distance on the earth's surface from the pole to the equator, now as the distance between two lines on a certain metallic rod preserved in the archives of the International Metric Commission at Paris.—*Standard Dictionary.*

The school should be provided with a set of metric measures and weights, a meter stick and a liter measure, and a carefully subdivided yardstick, or footrule, and a quart measure.

1. Memorize the—

TABLE FOR LINEAR MEASURE.

10 Mil'limeters (*mm.*)	= 1 Cen'timeter (*cm.*)
10 Centimeters	= 1 Dec'imeter (*dm.*)
10 Decimeters	= 1 ME'TER (*m.*)
10 Meters	= 1 Dek'ameter (*Dm.*)
10 Dekameters	= 1 Hek'tometer (*Hm.*)
10 Hektometers	= 1 Kil'ometer (*Km.*)
10 Kilometers	= 1 Myr'iameter (*Mm.*)

NOTE.—The kilometer is used in measuring long distances.

A 5-cent nickel piece is .02 meter in diameter, and .002 meters in thickness.

2. Write the table in the following form, filling the blanks:

1 millimeter = ... meter.
1 centimeter = ... meter.
1 decimeter = ... meter.
1 METER = UNIT OF LINEAR MEASURE.
1 dekameter = ... meters.
1 hektometer = ... meters.
1 kilometer = ... meters.
1 myriameter = ... meters.

1. Memorize the—

TABLE FOR SURFACE MEASURE.

100 sq. Millimeters (*sq. mm.*) = 1 sq. Centimeter (*sq. cm.*)
100 sq. Centimeters = 1 sq. Decimeter (*sq. dm.*)
100 sq. Decimeters = 1 sq. Meter (*sq. m.*)
100 sq. Meters = 1 sq. Dekameter (*sq. Dm.*)
100 sq. Dekameters = 1 sq. Hektometer (*sq. Hm.*)
100 sq. Hektometers = 1 sq. Kilometer (*sq. Km.*)
100 sq. Kilometers = 1 sq. Myriameter (*sq. Mm.*)

2. Memorize the—

TABLE FOR LAND MEASURE.

1 Cen'tare (*ca.*) = 1 Square Meter.
100 Centares = 1 ARE (*A.*)
100 Ares = 1 Hek'tare (*Ha.*)

3. Write the table in the following form, filling the blanks:

1 square millimeter = ... sq. meter.
1 square centimeter = ... sq. meter.
1 square decimeter = ... sq. meter.
1 SQUARE METER = UNIT OF SURFACE MEASUREMENT.
1 square dekameter = ... sq. meters.
1 square hektometer = ... sq. meters.
1 square kilometer = ... sq. meters.
1 centare = ... are.
1 hectare = ... ares.

4. How long in centimeters is one side of figure on page 76?
5. How long in decimeters is one side of figure on page 76?
6. How long in inches is one side of figure on page 76?
7. About how many square inches in a square decimeter?
8. About how many centimeters in an inch?
9. About how many square centimeters in a square inch?
10. About how many square decimeters on this page?
11. About how many square inches on this page?

1. Memorize the—

TABLE FOR SOLID MEASURE.

1000 cu. Millimeters (*cu. mm.*) = 1 cu. Centimeter (*cu. cm.*)
1000 cu. Centimeters = 1 cu. Decimeter (*cu. dm.*)
1000 cu. Decimeters = 1 cu. Meter (*cu. m.*)

For measuring wood, etc., a cubic meter is a stere, and—

10 Decisteres = 1 Stere (*st.*)
10 Steres = 1 Dekastere (*Ds.*)

2. Write the table in the following form, filling the blanks:

1 cu. millimeter = ... cu. meter.
1 cu. centimeter = ... cu. meter.
1 cu. decimeter = ... cu. meter.
1 CU. METER = UNIT FOR MEASURING VOLUME.
1 cu. meter = ... stere.
1 decistere = ... steres.
1 dekastere = ... steres.

3. Memorize the—

TABLE FOR CAPACITY.

10 Milliliters (*ml.*) = 1 Centiliter (*cl.*)
10 Centiliters = 1 Deciliter (*dl.*)
10 Deciliters = 1 LITER (*l.*)
10 Liters = 1 Dekaliter (*Dl.*)
10 Dekaliters = 1 Hektoliter (*Hl.*)
10 Hektoliters = 1 Kiloliter (*Kl.*)

4. Write the table in the following form, filling the blanks:

1 milliliter = ... liter.
1 centiliter = ... liter.
1 deciliter = ... liter.
1 LITER = UNIT FOR MEASURING CAPACITY.
1 dekaliter = ... liters.
1 hektoliter = ... liters.
1 kiloliter = ... liters.

The unit of capacity for measuring liquids, grains, etc., is the Liter, containing 1 cubic decimeter.

1. Memorize the—

TABLE FOR MEASURING WEIGHT.

10 Milligrams (*mgs.*)	= 1 Centigram (*cg.*)
10 Centigrams	= 1 Decigram (*dg.*)
10 Decigrams	= 1 Gram (*g.*)
10 Grams	= 1 Dekagram (*Dg.*)
10 Dekagrams	= 1 Hektogram (*Hg.*)
10 Hektograms	= 1 Kilogram (*Kg.*)
10 Kilograms or Kilos	= 1 Myriagram (*Mg.*)
10 Myriagrams	= 1 Quintal (*Q.*)
10 Quintals	= 1 Tonneau (*T.*)

A cubic centimeter of water, at its greatest density (39.2° F. or 4° C.) weighs 1 *gram*. It is used in weighing very small articles, the *kilogram* is used in common trade, and the *ton* in weighing very heavy articles.

2. Write the table in the following form, filling the blanks:

1 milligram	= ... gram.
1 centigram	= ... gram.
1 decigram	= ... gram.
1 GRAM	= UNIT FOR MEASURING WEIGHT.
1 dekagram	= ... grams.
1 hektogram	= ... grams.
1 kilogram, or 1 KILO	= ... grams.
1 myriagram	= ... kilos.
1 quintal	= ... kilos.
1 tonneau	= ... kilos.

A nickel 5-cent piece weighs 5 grams.

A silver dollar weighs 25 grams.

3. What is the weight of 12 Hl. of water in kilos?

4. What is the weight of 500 silver dollars?

5. Compare a cubic meter of water with 1 tonneau.

6. What is the weight of 450 cu. cm. of alcohol, if it is 80% as heavy as water?

1. Find the dimensions of this page.
2. Find the dimensions of the desk.
3. Find the dimensions of the blackboard.
4. Find the dimensions of the schoolroom.
5. Find the area of this page in sq. cm.
6. Find the area of the top of the desk in sq. dm.
7. Find the area of the schoolroom floor in sq. m.
8. Find the area of the blackboard in sq. m.
9. Find the volume of your schoolroom.
10. Find the value of a pile of wood 30 m. long, 1.5 m. wide, and 2.5 m. high, at $.75 a stere.
11. How many liters in a vessel 40 cm. by 35 cm. by 75 cm.?
12. A vessel 1 m. square contains water 15 cm. deep. How many liters of water in it?
13. How many hektoliters of grain in a bin 3 m. square and 2 m. high?
14. A dealer bought a quintal of butter for $88, and sold it for $1.10 a kilogram. What per cent did he make?
15. How many hectoliters of potatoes, at 20 cents per dekaliter, are required to pay for 3 kilograms 4 hektograms of tea, at $2.40 per Kg.?
16. How many kilometers of wire fence will be required to surround a square field 350 meters on each side with a fence four wires high?
17. From 12 decasteres of wood were sold 10 steres. Find the value of what remained, at $1.50 per stere.
18. How much carpet .8 of a meter wide is required to carpet a room 10.4 meters long and 8.4 meters wide?
19. How long must a pile of wood be to contain 48 steres, provided it is 4 meters high and 3 meters wide?
20. What would be the cost of a pile of wood 15.7 m. long 3 m. high, and 7.52 m. wide, at $1.50 a stere?
21. What would be the cost of excavating a cellar 18.3 m. long, 10.73 m. wide and 3.4 m. deep, at 15 cents per stere?

THE FOLLOWING METRIC EQUIVALENTS of measures and weights have been established by Congress for use in legal proceedings. It is also adopted by the United States coast survey, and is used partially in the postoffice department and mint, and extensively in the arts and sciences.

LINEAR MEASURE.

1 centimeter=.3937 in.	1 in. =2.54 centimeters.
1 decimeter =3.937 in.=.328 ft.	1 ft. =3.048 decimeters.
1 meter =39.37 in.=1.0936 yds.	1 yd.=.9144 meter.
1 dekameter=1.9884 rds.	1 rd. =.5029 dekameter.
1 kilometer =.62137 mi.	1 mi.=1.6093 kilometers.

SQUARE MEASURE.

1 sq. centimeter=.1550 sq. in.	1 sq. in.=6.452 sq. centimeters.
1 sq. decimeter =.1076 sq. ft.	1 sq. ft.=9.2903 sq. decimeters.
1 sq. meter =1.196 sq. yd.	1 sq. yd.=.8361 sq. meter.
1 are =3.954 sq. rds.	1 sq. rd.=.2529 are.
1 hektare =2.47 acres	1 acre =.4047 hektare.
1 sq. kilometer =.386 sq. mi.	1 sq. mi.=2.59 sq. kilometers.

1. How many yards, feet, and inches in 10.5 meters?

2. How many yards, feet, and inches in 17.75 meters?

3. How many ares of land in a quarter section?

4. How many hektares of land in $2\frac{1}{2}$ sections?

5. How many feet and inches in 5 dekameters?

6. How many rods are in $1\frac{2}{5}$ hektometers?

7. How many square rods in $\frac{3}{5}$ of a hektare?

8. How many yards of carpet, $\frac{3}{4}$ of a yard wide, will cover a floor 15 feet wide and 18 feet long? Change the dimensions to meters, and solve.

9. A field is 24 rods long and 20 rods wide. Change the dimensions to meters, and express the area in hektares.

10. The top of a table was 2 feet wide and $2\frac{1}{2}$ feet long. Change the dimensions to meters, and express the area in square meters.

11. What is the area in square meters of a lot 25 feet by 100 feet?

VOLUME AND CAPACITY.

1 cubic centimeter	=	.061	cubic inch.
1 cubic decimeter	=	.0353	cubic foot.
1 cubic meter, 1 stere	=	1.308	cubic yards.
		.2759	cord.
1 liter	=	.908	quart (dry).
		1.0567	quarts (liquid).
1 dekaliter	=	2.6417	gallons.
		1.135	pecks.
1 hektoliter	=	2.8375	bushels.
		26.42	gallons.
1 cubic inch	=	16.39	cubic centimeters.
1 cubic foot	=	28.317	cubic decimeters.
1 cubic yard	=	.7646	cubic meter.
1 cord	=	3.624	steres.
1 quart (dry)	=	1.101	liters.
1 quart (liquid)	=	.9463	liter.
1 gallon	=	.37885	dekaliter.
1 peck	=	.821	dekaliter.
1 bushel	=	.3524	hektoliter.

WEIGHTS.

1 gram	=15.432 gr. (Tr.)	1 ounce	=28.35 grams.
1 kilogram	=2.2046 lbs.(Av.)	1 lb. (Av.)	=.4536 kilogram.
1 metric ton	=2204.6 lbs.(Av.)	1 ton(2000 lbs.)	=.9072 metric ton.

APPROXIMATE EQUIVALENTS.

1 decimeter	=4 inches.	1 liter	= 1.06 qt. liquid.
			.9 qt. dry.
1 meter	=1.1 yards.		
1 kilometer	=$\frac{5}{8}$ of a mile.	1 hektoliter	= $2\frac{5}{8}$ bushels.
1 hektare	=$2\frac{1}{2}$ acres	1 kilogram	= $2\frac{1}{5}$ pounds.
1 stere, cu. m.	=$\frac{1}{4}$ cord.	1 metric ton	= 2200 pounds.

1. How many liters in a bushel of wheat?
2. How many steres in $10\frac{1}{2}$ cords of wood?
3. Change 10 gallons of milk to liters.
4. Change $5\frac{1}{2}$ pecks of grain to deciliters.

1. Having the length of a meter in inches, how are the equivalents derived for the other denomination of metric linear measure?

2. If one centimeter equals .3937 of an inch, how can be found the equivalent of 1 inch in centimeters?

3. Having the area of a sq. m. in square yards, how is found the equivalent of 1 sq. dm. in feet?

4. If 1 sq. meter equals 1.196 sq. yards, how can be found the value of 1 sq. yard in meters?

5. If 1 liter equals .908 of a quart, how find the number of liters in a bushel?

6. If 1 hektare equals 2.47 acres, how can be found the value of 1 acre in hektares?

7. If 1 stere equals .2759 of a cord, how can be found the value of 1 cord in steres?

8. If 1 dry quart equals 1.101 liters, how many hektoliters in a bushel?

9. If 1 cu. centimeter equals .061 of a cubic inch, how many cubic centimeters in 1 cubic inch?

10. If 1 are equals 3.954 square rods, how find the equivalent of 1 square meter in square yards?

11. If 1 metric tonneau equals 2204.6 lbs., how find the value of 1 ton in metric tonneaus?

12. Beginning with the value of 1 cubic meter in cubic inches, derive the other equivalents for measuring volume and capacity.

13. Beginning with the weight of 1 gram in grains, derive the other equivalents for measuring weights.

14. Which is cheaper and how much, to buy cloth at $3 per meter, or at $2.90 per yard?

15. In 12345.6 pounds, how many kilograms?

16. How many hecktares in 4,784 square yards?

17. How many acres in a rectangular field 800 meters long and 200 meters wide?

18. Make an original problem involving metric units.

MENSURATION.

LUMBER is the name given to boards, planks, scantling, joists, and other sawed timber.

A BOARD FOOT is the unit of measurement. It consists of a board 1 foot square, and 1 inch thick, It contains, therefore, 144 cubic inches.

In calculating the measurement of boards, it is convenient to consider the board 12 feet long. Since a board 12 feet long and 1 inch wide contains 1 board foot, (144 cubic inches) it will contain as many board feet as it is inches wide. Thus, a 12-foot board 2 inches wide contains 2 board feet; one four inches wide contains 4 board feet, etc. If a board is less than 1 inch thick, it is considered as if it were 1 inch thick. If lumber is more than 1 inch in thickness, the amount is found as in 1-inch lumber, and then by multiplying by the number of inches in the thickness.

1. How many feet of lumber in 5 12-foot boards, having a width respectively, 12, 10, 18, 7, and 9 inches?

2. How many feet of lumber in 6 12-foot boards, 6, 8, 9, 10, 12, and 15 inches wide, respectively?

If the boards are 14 feet long, find the board feet for 12-foot boards and add $\frac{1}{6}$ of the result to itself; the 2 feet over 12 is $\frac{1}{6}$ of 12. In like manner, to find the amount of lumber in 15-foot boards, find it for 12-foot boards and add $\frac{1}{4}$ of the result; for 16-foot boards, add $\frac{1}{3}$ of the result; for 18-foot boards, add $\frac{1}{2}$ of the result; for 10-foot boards, subtract $\frac{1}{6}$ of the result; for 9-foot boards, subtract $\frac{1}{4}$ of the result, etc.

3. How many feet of lumber in 5 14-foot boards, 12, 10, 14, 8, and 11 inches wide, respectively?

4. How many feet of lumber in 6 boards, 8, 9, 10, 12, 14, and 15 inches wide, respectively, all being 16 feet long?

5. How many feet of lumber in 5 boards, 18 feet long and 9, 10, 12, 14, and 15 inches wide, respectively?

6. What would the answer to the preceding problem be if the board were 10 feet long?

In case the dimensions of lumber are such that the calculation is difficult by the methods suggested above, it may be observed that to *multiply the length in feet by the width and thickness in inches and to divide the product by 12*, will give the board feet for a result.

1. At \$18 per M., find the cost—

Of 20 scantling, 15 feet long, 4x4 inches.
Of 20 joists, 16 feet long, 2x8 inches.
Of 20 joists, 18 feet long, 2x8 inches.
Of 10 pieces, 18 feet long, 4x8 inches.
Of 12 pieces, 16 feet long, 8x8 inches.
Of 26 pieces, 12 feet long, 4x6 inches.
Of 47 pieces, 10 feet long, 2x12 inches.

2. How many boards 12 feet long 6 inches wide are required for a floor 36 feet by 27 feet?

3. What will be the width of a board 1 inch thick 18 feet long, and containing 21 board feet?

4. At \$20 per M., find the cost of lumber for a 6-foot sidewalk around a block 200 feet square, with plank 2 inches thick, laid on 2 stringers 2x4, placed at the inner and outer edges of the walk.

NOTE.—**Stringers are the timbers lengthwise, to which the boards are nailed.**

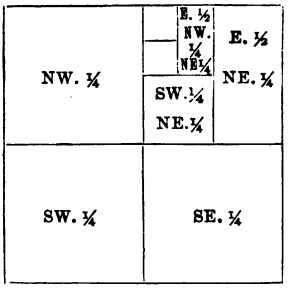
5. At \$10 per M., what will be the cost of lumber for a tight board fence around a block 240 feet square, posts 8 feet apart, stringers 2x4, fence to be 4 feet high, and posts to cost 16 cents each?

6. Find the cost of the lumber for a common board fence 4 boards high, around the same lot, the posts to be 12 feet apart, and costing 16 cents apiece.

NOTE.—**Common flooring and fencing is 6 inches wide, unless otherwise indicated.**

7. How much inch lumber will be required to cover a walk 4 feet wide around a rectangular garden, 300 yards by 200 yards?

8. Find the cost of flooring for two rooms, each 24x20½ feet, with boards 1¼ inches thick, at \$20 per M.



A SECTION of land is one mile square and contains 640 acres.

A QUARTER SECTION is one half a mile square and contains 160 acres.

A QUARTER of a QUARTER of a section is one fourth of a mile square and contains 40 acres.

A QUARTER of a QUARTER of a QUARTER of a section is one eighth of a mile square and contains 10 acres.

1. Draw a diagram illustrating the section and the fractions of a section mentioned above.

2. My farm consists of the NE. $\frac{1}{4}$ (northeast quarter) and the N. $\frac{1}{2}$ of the SE. $\frac{1}{4}$ of section 21. Draw a diagram of the farm. How many acres in it? How many rods of fence required to enclose it?

3. A farm consists of the SW. $\frac{1}{4}$ of the SE. $\frac{1}{4}$, and the E. $\frac{1}{2}$ of the SE. $\frac{1}{4}$ of the SW. $\frac{1}{4}$ of section 21. Make a diagram of the section and the farm. How many acres in the farm? How many rods of fence required to enclose it?

4. How many rods required to enclose a section? A half section? A quarter section? A quarter of a quarter of a section?

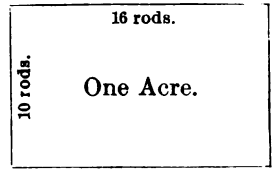
5. How many rods required to enclose ten acres off one side of a quarter of a quarter of a section?

6. How many rods required to enclose a square ten-acre lot?

7. Are four times as many rods required to enclose forty acres as are required to enclose ten acres? Prove by diagram.

AN ACRE contains 160 square rods.

It may be 1 rod by 160 rods.
2 rods by 80 rods.
4 rods by 40 rods.
8 rods by 20 rods.
16 rods by 10 rods.
32 rods by 5 rods, etc.



1. How many acres in a field 40 rods by 16 rods? 32 rods by 25 rods? 160x17 rods? 64x10 rods? 12x40 rods?

2. How many acres in a piece of land 1 mile by 4 rods? $\frac{1}{2}$ mile by 11 rods? $\frac{1}{4}$ mile by 20 rods? $\frac{1}{8}$ mile by 16 rods? $\frac{1}{10}$ mile by 25 rods?

A SURVEYOR'S CHAIN is 4 rods (66 ft.) long. It is divided into 100 LINKS. Chains and links may be expressed as chains and hundredths of chains, as 4 chains 15 links are 4.15 chains.

3. Express in acres, 1 square chain; 1 ch. by 2 ch.; 2 ch. by 3 ch.; 5 ch. by 4 ch.; 6 ch. by 8 ch.; 5 ch. by 9 ch.

4. How many acres in a piece of land 6 chains 20 links by 4 chains 25 links? 6 chains 15 links by 6 chains 50 links?

5. How many acres in a field 48 rods by 28 rods?

SOLUTION.

48 rods = 12 chains.
28 rods = 7 chains.
7x12 = 84 sq. chains.
10 sq. chains = 1 acre.
84 sq. chains = 8.4 acres.

How many acres in a field—

6. 50 rods by 40 rods?
7. 36 rods by 40 rods?
8. 25 rods by 60 rods?
9. 80 rods by 36 rods?
10. 48 rods by 50 rods?
11. 75 rods by 32 rods?
12. 24 chains by 10 chains?
13. 36 chains by 25 chains?
14. 96 chains by 65 chains?
15. 50 chains by 45 chains?
16. 40 chains by 70 chains?
17. 75 chains by 96 chains?

CARPET is sold by the *yard in length*. In finding the yards required for a room, consideration must be given to the *direction* the strips are laid across the room, and the *number* of strips required, and the allowance for *matching* the patterns or figures. If a fraction of a strip is required, a whole strip is purchased and the part not used is turned under.

1. In a room 13 feet 8 inches wide and 14 feet 6 inches long, how many strips of carpet 1 yard wide are required, the strips to run the short way? How much to be turned under? How many strips required to run the long way? How much to be turned under? If there is no waste in matching, how many yards required, if the strips run the short way? How many if they run the long way?

2. A room 15 feet by 16 feet 4 inches is to be covered with carpet $\frac{3}{4}$ of a yard wide. How shall it be laid that the least is turned under? How many yards are required, if there is a waste of 6 inches in matching the figure on each strip, except the first? Explain why there is no waste on the first strip?

3. A room 14 feet 3 inches by $16\frac{1}{2}$ feet is to be covered with carpet 1 yard wide. If the strips run the long way, how many are required? How much turned under? If the figure in the carpet is 12 inches long, how much must be allowed in matching? How many yards are required?

4. Find the cost of carpet $\frac{3}{4}$ of a yard wide, at \$2.75 per yard, for a room 34 feet 8 inches by 13 feet 3 inches, if there is a waste of $\frac{1}{4}$ of a yard in matching on each strip except the first.

NOTE.—**Strips are laid the long way unless otherwise indicated.**

5. Which way must the strips of carpet $\frac{3}{4}$ of a yard wide run in order to carpet at the least cost a floor 20 feet 6 inches long and 19 feet 6 inches wide, if there is no waste for matching?

6. Find the cost of carpeting a floor 34 feet by 18 feet 6 inches, carpet $\frac{3}{4}$ of a yard wide, at \$.90 a yard.

7. Find the cost of a floor 30 feet 3 inches by 22 feet, carpet $\frac{3}{4}$ of a yard wide, at \$1.08 a yard.

WALL PAPER is sold by the roll, each roll being 8 yards long and ½ yard wide. It is also made in double rolls 16 yards long.

It is impossible to make an exact estimate of the amount of paper required to cover the walls of a room, on acccount of the waste in matching the pattern or in cutting the strips from the roll.

An *approximate estimate* is made by finding the number of yards along the wall or around the room to be covered, and taking twice the number for the number of strips required. The length of the strip determines the number which can be cut from a roll or double roll. For a part of a strip a whole strip is estimated, and for a part of a roll a whole roll is estimated.

Imported paper of other dimensions than the size given above is estimated in the same manner, taking into consideration the width of the paper and length of the roll.

Allowance may be made for doors and windows.

1. If a wall is 7 feet high, which will make the less waste, a single roll or a double roll? If the wall is 9 feet high? If it is 6½ feet high?

2. If a wall is ten feet high and has a base board 1 foot wide, and a border 1 foot wide, how long will be the strips of paper on it?

3. If a room is 9 feet wide, how many strips of paper will cover one end?

4. How many strips are required to cover the walls of a room 12 feet by 19 feet?

5. How many strips of paper are required to cover the walls of a room 10 feet square?

6. How many rolls of paper required to cover a wall 18 feet long and 8 feet high?

7. How many rolls required to cover a wall 24 feet long and 15 feet high?

NOTE.—Are single rolls or double rolls the better?

8. How many rolls of paper required to cover the walls of a room 19 feet by 15 feet and 9 feet high above the base board?

ORDINARY SHINGLES are 4 inches wide, and are laid 4 inches "to the weather." Each shingle covers 16 square inches, and 9 shingles cover 1 square foot. Since some are wasted in laying, it is customary to estimate the number required by the "square" of 100 square feet, allowing 1,000 shingles to each "square," thus allowing 100 shingles for waste for each "square."

Shingles are packed in bunches of 250; hence it requires 4 bunches for each "square" of roofing. A full bunch is sold when a part of a bunch is required.

1. What is the *exact* number of shingles required for a "square" of roofing, when laid 4 inches "to the weather?"

2. Find the exact number of shingles required for a "square," when laid 6 inches "to the weather."

3. Estimate the number of shingles required for a double roof 30 feet long, with rafters 20 feet long.

NOTE.—Always estimate for the lower course on each side of the roof being laid double.

4. Estimate the shingles required for a roof 40 feet long, with rafters 25 feet long.

5. Find the cost at $3.50 per M. of shingles for a double roof 50 feet long, with rafters 24 feet long.

6. Find the difference in cost of roofing a shed 35 feet long, rafters 16 feet long, with shingles at $.75 a bunch, and with inch lumber laid double, at $12 per M.

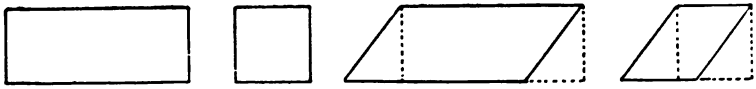
7. How many feet of lumber in 4x5 rafters placed 30 inches apart in a double roof 40 feet long, the rafters being 20 feet long?

8. How many feet in a piece of timber 45 feet long and 25 inches square?

9. How much lumber is required to build a fence around a field 80 rods square, the fence to be four boards high, the posts to be 6 feet long and 4 inches square, and set 8 feet apart?

10. How much lumber is required to sheet a roof for shingling a barn 40 feet long, the rafters on each side being 20 feet long, and the boards projecting 1 foot at each end of the building?

QUADRILATERALS.



Rectangle. Square. Rhomboid. Rhombus.

A QUADRILATERAL is a figure having four sides and therefore four angles.

A PARALLELOGRAM is a quadrilateral whose opposite sides are parallel. The *altitude* is the perpendicular distance between its opposite sides.

A parallelogram which is right-angled is called a *Rectangle*. When the four sides of a rectangle are equal, it is called a *Square*. To find the area of a rectangle, see page 165, Book One.

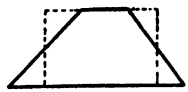
An oblique-angled parallelogram is called a *Rhomboid*. An equilateral rhomboid is called a *Rhombus*.

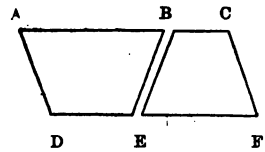
Cut from paper a rhomboid, and change it as indicated in the figure to show that its area is the same as the area of a rectangle having the same base and altitude.

Draw the figure * and find the area—

1. Of a parallelogram 20 feet long and 18 feet wide.
2. Of a parallelogram 30 feet long and 12 feet wide.
3. Of a parallelogram with a base 36 feet and altitude 15 feet.
4. Of a parallelogram with a base 16 feet and altitude 12 feet.
5. Of a parallelogram with a base 64 feet and altitude 24½ feet.
6. Of a parallelogram with a base 40 rods and altitude 18 rods.
7. Find the difference in the area of two floors, one 36 feet by 15 feet, and the other 40 feet by 20 feet. What is the largest unit of surface that will measure both floors? What per cent is the first of the second?
8. A field in the form of a rhombus is 80 rods on each side, and contains 16 acres. What is the width of the field? Draw a diagram of it.
9. Write the rule for finding the area of a parallelogram.

* Draw all lines proportionate to the dimensions given.





A TRAPEZOID is a quadrilateral having only two of its sides parallel.

Cut from paper a trapezoid as indicated in the figure, and show that its area is the area of a rectangle having the same width, with a length found by averaging the two parallel sides.

NOTE.—The average is found by adding the two sides and dividing by two.

Cut from paper two similar trapezoids, and place them as indicated in the figure. What is the resulting figure? Are the lines A C and D F equal? Why? Compare the area of the entire figure with one of the trapezoids. If the width of the trapezoid and the length of the two parallel sides are given, what method is suggested by the figure of finding the area of the trapezoid?

1. Find the area in board feet of a board 12 feet long, 16 inches wide at one end and 8 inches at the other.

2. The parallel sides of a field are 40 chains and 23 chains, and the width is 25 chains. Find the number of acres in the field.

3. The parallel sides of a field are 75 rods and 100 rods, and the width is 60 rods. Find the number of acres in the field.

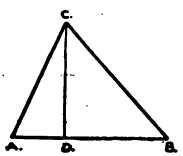
4. The parallel sides of a trapezoid field are 130 yards and 222 yards, and the field contains 8 acres. What is the distance between the parallel sides?

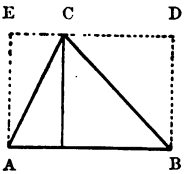
5. Find the area in square feet of a board 16 feet long, 30 inches wide at one end and coming to a point at the other.

6. The parallel sides of a trapezoid are 87 inches and 63 inches, and the altitude is 15 inches. Find the area in square feet.

7. The parallel sides of a trapezoid are 2 feet 2 inches and 3 feet 3 inches; the altitude is 1 foot 4 inches. Find the area.

TRIANGLES.

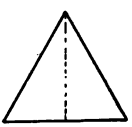




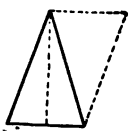
A TRIANGLE is a plane figure bounded by three straight lines.

The *Base* of a triangle is the line upon which it stands; and the *Altitude* is the height of its vertex above the base, or the base extended. Thus, B A is the base, and C D the altitude of the triangle.

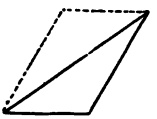
Draw the lines A E and D B perpendicular to the extremities of the base of the triangle A C B, and draw the line E D through C, parallel to A B. What is the resulting figure? Tell how it was made. What part of the whole figure is the triangle?



Equilateral



Isosceles



Scalene

AN EQUILATERAL TRIANGLE is a triangle that has all of its sides equal. How is it changed into a rhomboid of equal area? How is it changed into a rectangle of equal area?

AN ISOSCELES TRIANGLE is a triangle that has two of its sides equal.

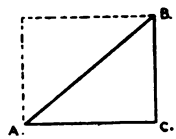
Draw the figure as indicated by the dotted lines. What is the resulting figure? What part of the whole figure is the triangle?

A SCALENE TRIANGLE is a triangle that has no two sides equal.

Draw the figure as indicated by the dotted lines. What is the resulting figure? What part of the whole figure is the triangle?

Is an equilateral triangle an isosceles triangle?

How is an isosceles triangle divided into two equal right-angled triangles?



A RIGHT-ANGLED TRIANGLE is a triangle that has one right angle. Thus, A C B is a right-angled triangle because the angle A C B is a right angle. The side A B, opposite the right angle is the *hypotenuse*, the side A C the *base*, and the side B C the *altitude*.

Complete the parallelogram as indicated in the figure. What part of the parallelogram is the triangle? From the method of finding the area of a parallelogram, what method is suggested for finding the area of a triangle?

Find the area of a triangle having—

1. A base of 40 feet and an altitude of 36 feet.

2. A base of 30 rods and an altitude of 24 rods.

3. A base of 46 ft. 3 in. and an altitude of 25 ft. 6 in.

4. A base of 75 yards and an altitude of 44 yards.

5. A base of 25 rods and an altitude of 165 feet.

6. The gable ends of a barn are 30 feet wide, and the ridge is $7\frac{1}{2}$ feet above the eaves. How many feet of lumber is required to cover both gables?

7. How many acres in a triangular field the base of which is 95 chains and the altitude 60 chains?

8. The area of a triangle is $140\frac{5}{8}$ square feet; the altitude is 150 inches. Find the base.

9. Find the altitude of a triangle whose area is 6 square feet and base 27 inches.

10. Find the base of a triangle whose area is 20 acres and altitude 20 rods.

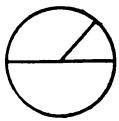
11. Find the base of a triangle whose area is 50 square yards and altitude 40 yards.

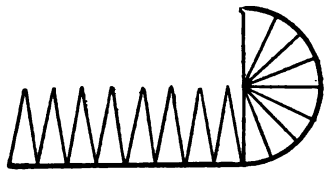
12. Find the area of a triangle with base 4 yards 2 feet, altitude, 3 rods 2 feet.

13. Write the rule for finding the area of a triangle when the base and altitude are given.

14. How find the base, when the area and altitude are given?

CIRCLES.





A CIRCLE is a plane figure bounded by a curve line, every point of which is equally distant from a point within called the *center*.

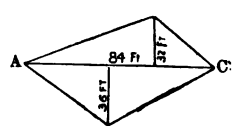
The curve line is called the *circumference*, and a straight line passing through the center and ending in the circumference is the *diameter*. Half the diameter is called the *radius*.

Measure the circumference of a plate, and compare it with the diameter. Measure also in the same manner a bucket, stove-pipe, a bicycle wheel, a silver dollar and other circular objects. It will be found that the circumference of a circle is a little more than three times the length of the diameter.

Any circle may be supposed to be divided into equal triangles. If the whole circle be straightened out, what part of the circle corresponds to the bases of the triangles? What part corresponds to the altitude of the triangles? What method is suggested for finding the area?

In geometry it is found that a circumference of a circle is 3.1416 times the diameter. When the diameter is given, how is the circumference found? When the circumference is given, how is the diameter found?

A TRAPEZIUM is a quadrilateral having no two of its sides parallel.



A DIAGONAL is a straight line joining any two angles of a plane figure not adjacent, as the line *A C*.

It will be seen from the above diagram that a diagonal divides a trapezium into two triangles.

1. Find the circumference of a circle, having a diameter of 30 inches.

2. Find the circumference of a circle, having a diameter of 36 feet.

3. Find the circumference of a circle, having a diameter of 45 feet.

4. Find the circumference of a circle, having a diameter of 48 feet.

5. Find the diameter of a circle, having a circumference of 15 feet.

6. Find the diameter of a circle, having a circumference of 19 feet.

7. Find the diameter of a circle, having a circumference of 57 rods.

8. Find the diameter of a circle, having a circumference of 33 yards.

9. Find the diameter of a circle, having a circumference of 78.54 feet.

10. Find the area of a circle 186 feet in diameter.

11. Find the area of a circle $\frac{1}{5}$ of a mile in diameter.

12. Find the area of a circle having a radius of $37\frac{1}{2}$ feet.

13. Find the acres in a circular field one mile in circumference.

14. Find the area on which a horse can graze when tied to a stake by a rope 1 chain in length.

15. A barn 40 feet by 30 feet stood in a pasture field. A cow was tied to one corner by a rope 30 feet long. Find the area on which she could graze. If the rope had been 40 feet long, what area could she have grazed?

16. Two circles 24 feet and 40 feet in diameter have the same center. Find the area of the surface between the circumferences.

17. Measure the diameter of a bicycle wheel, and find the number of revolutions it will make in going about a circle 1 mile in diameter.

18. The diameter of a circle is how many times the radius?

RATIO AND PROPORTION.

Two numbers of the same kind may be compared—

1. By considering *how much* one is greater or less than the other, which is shown by their difference.

2. By considering *how many* times one number is greater or less than another, which is shown by their quotient.

In comparing two numbers by means of their difference, the less is always taken from the greater.

In comparing two numbers by their quotient, one is regarded as a *standard* which *measures* the other; hence, to measure a number, is to find how many times it contains the *standard*.

RATIO.

A RATIO is the relative magnitude of two like numbers and is the quotient arising from dividing the first by the second.

The ratio of one number to another is shown in two ways:

1. By a colon: thus, 16 : 4; and is read, 16 is to 4; or, 16 divided by 4.

2. In a fractional form, as $\frac{16}{4}$; read, 16 divided by 4.

THE TERMS of a ratio are the two numbers compared. The *Antecedent* is the first term of a ratio, the *Consequent* is the second term, and the two terms together are called a *Couplet*.

AN INVERSE RATIO is a ratio formed by inverting the terms of a given ratio. Thus,

8 : 9 is the inverse of 9 : 8.

A SIMPLE RATIO is the ratio of two numbers. Thus,

21 : 3 = 7, is a simple ratio.

A COMPOUND RATIO is the product of two or more simple ratios. It is usually indicated by means of the brace. Thus,

$$\left.\begin{matrix} 8:2 \\ 6:2 \\ 10:5 \end{matrix}\right\}$$, or $8\times6\times10 : 2\times2\times5$ is a compound ratio, equal to the simple ratio 480 : 20.

A compound ratio may be changed to a simple ratio by *multiplying antecedents together for a new antecedent, and consequents for a new consequent.*

PROPORTION.

PROPORTION is the expression of equality of ratios. Thus, the ratio of 6 : 3 and 8 : 4 are equal, and if written thus, 6 : 3 = 8 : 4, we have a proportion.

The equality of ratios is generally indicated by writing a double colon between the two ratios; thus, 6 : 3 :: 8 : 4.

A proportion is read in several different ways. Thus, 6 : 3 :: 8 : 4 may be read: (1) *The ratio of* 6 *to* 3 *equals the ratio of* 8 *to* 4; (2) 6 *is to* 3 *as* 8 *is to* 4.

A proportion consists of four terms. The first and fourth are called the *extremes*, the second and third the *means*.

The first and second terms together are called the *first couplet*, the third and fourth the *second couplet*.

Each term of a proportion is called a *Proportional;* and when the two means are the same number, that number is a *Mean Proportional* between the two extremes. Thus, in 12 : 6 :: 6 : 3, 6 is a mean proportional between 12 and 3.

Proportions may be SIMPLE or COMPOUND. A Simple Proportion is the comparison of simple ratios. A Compound Proportion is the comparison of compound ratios.

A SIMPLE PROPORTION applies to the solution of questions in which three terms of a proportion are given to find a fourth. Of the given terms *two must be of the same kind*, and constitute a ratio; and the other must be of the *same kind as the required term*, and constitute with it another ratio equal to the first.

The *solution of a proportion* is the process of finding one of its terms, when the other three are given.

Find the ratio of—

1. 16 to 4 ; 21 to 7.
2. 18 to 6 ; 36 to 18.
3. 16 to 8 ; 48 to 16.
4. 45 to 15 ; 60 to 15.
5. $2\frac{1}{4}$ to 10 ; $3\frac{1}{8}$ to 25.
6. 100 to $12\frac{1}{2}$; $6\frac{1}{4}$ to 50.
7. 300 to 60 ; 60 to 240.
8. 70 to 210 ; 210 to 30.
9. 27 to 9 ; 81 to 27.
10. $10\frac{1}{2}$ to $3\frac{1}{2}$; $7\frac{1}{2}$ to $37\frac{1}{2}$.
11. $8\frac{1}{3}$ to $33\frac{1}{3}$; $12\frac{1}{2}$ to $87\frac{1}{2}$.
12. $\frac{1}{2}$ pk. to $\frac{1}{2}$ bu.; 4^2 to 3^2.

Find the ratio when—

1. The consequent is 26 and the antecedent is 13.
2. The antecedent is 9 and the consequent is 36.
3. The consequent is 7 and the antecedent is 28.
4. The consequent is 12 and the antecedent is 24.
5. The consequent is 33 and the antecedent is 11.
6. The antecedent is 15 and the consequent is 75.
7. The consequent is $\frac{1}{8}$ and the antecedent is $\frac{1}{10}$.

Find the antecedent when—

8. The consequent is 40 and the ratio is 5.
9. The consequent is $\frac{2}{3}$ and the ratio is $\frac{1}{2}$.
10. The ratio is $\frac{1}{2}$ and the consequent is 10.

Find the consequent when—

11. The antecedent is 9 and the ratio is 7.
12. The antecedent is $\frac{1}{2}$ and the ratio is 3.
13. Find the missing term in the proportion, 15 : 90 :: 30 : x.

SOLUTION.

The consequent 90 is 6 times the antecedent 15; hence,
The consequent desired is 6 times the antecedent 30, or 180.

Or,

The antecedent 30 is twice the antecedent 15, hence,
The consequent desired is twice the consequent 90, or 180.

14. Find the missing term in the proportion 90 : x :: 120 : 360.

SOLUTION.

The consequent 360 is 3 times the antecedent 120, hence,
The consequent desired is 3 times the antecedent 90, or 270.

Or,

The antecedent 90 is $\frac{3}{4}$ of the antecedent 120, hence,
The consequent desired is $\frac{3}{4}$ the consequent 360, or 270.

15. Find the missing term in the proportion \$50 : \$150 :: x : 270.

SOLUTION.

The antecedent \$50 is $\frac{1}{3}$ the consequent \$150;
The antecedent desired is $\frac{1}{3}$ of 270, or 90.

Find the missing term—

1. 6 : 18 :: 7 : x.
2. 3 : 12 :: 10 : x.
3. x : 10 :: 6 : 30.
4. x : 30 :: 8 : 48.
5. 6 : 18 :: x : 45.
6. 8 : x :: 7 : 42.
7. 4 : 16 :: 20 : x.
8. 3 : 7 :: x : 28.
9. x : 9 :: 12 : 27.
10. 10 : x :: 7 : 35.
11. 6 : 14 :: x : 70.
12. 11 : 55 :: 9 : x.
13. x : 14 :: 36 : 18.
14. 6 : 9 :: x : 12.
15. 12 : 16 :: 48 : x.
16. 36 : 48 :: 48 : x.
17. \$60 : \$33 :: 20 yds. : x.
18. x : \$16 :: 80 yds. : 8 yds.
19. x : \$18 :: 63 yds. : 42.
20. 24 : x :: 42 : 28.
21. 20 : 30 :: 30 : x.
22. x : 24 :: 130 : 13.
23. x : 20 :: 13 : 130.
24. 30 : x :: 45 : 18.
25. 14 : x :: 21 : 105.
26. 18 : 54 :: 54 : x.
27. \$300 : \$60 :: x : 50.
28. 500 : x :: 125 : 625.
29. x : 100 :: 30 : 40.
30. $\frac{1}{2}$: $\frac{3}{4}$ :: x : 3.
31. .5 : 1 :: 75 : x.
32. $12\frac{1}{2}$: $6\frac{1}{4}$ :: \$.20 : x.
33. $6\frac{1}{4}$: x :: 100 : 800.
34. $2\frac{1}{4}$: $\frac{3}{4}$ :: 60 : x.
35. 2 : 2 :: x : 6.
36. $\frac{5}{8}$: $6\frac{2}{3}$ :: 100 : x.
37. 5 doz. : $\frac{1}{2}$ doz. :: x : \$.75.
38. \$.08$\frac{1}{3}$: \$.50 :: x : 12 pts.
39. x : 12 bu. :: \$3.60 : \$7.20.
40. 3 yds. : 12 yds. :: x : \$21.
41. x : 16 yds. :: \$5.25 : \$21.
42. 96 : x :: \$67$\frac{1}{4}$: \$201$\frac{3}{4}$.
43. $1\frac{1}{2}$ lbs. : $\frac{3}{4}$ lb. :: \$1 : x.
44. $2\frac{1}{4}$ lbs. : x oz. :: \$2 : \$.25.
45. x : \$16 :: 8 bu. 2 pks. : 34 bu.
46. 2 bu. 3 pks. : x :: \$3.50 : \$10.50.
47. 1 hr. 12 min. : x :: 32 : 8.
48. $\frac{1}{4}$: $\frac{5}{8}$ :: x : 30.
49. \$1 : x :: $\frac{5}{8}$: $\frac{1}{4}$
50. \$28.50 : \$9.50 :: x : $4\frac{1}{2}$ yds.
51. $6\frac{1}{4}$: $37\frac{1}{2}$ :: 100 : x.
52. $37\frac{1}{2}$: $4\frac{1}{6}$ :: x : \$27.
53. $\frac{2}{3}$: $\frac{3}{4}$ :: x : 45.
54. x : $\frac{6}{7}$ :: $\frac{1}{8}$: $\frac{3}{4}$.
55. .05 : 50 :: 10 : x.
56. .4 : $33\frac{1}{3}$ :: x : 25.
57. 36 : x :: { 4 : 8. / 8 : 9.
58. 15 : 60 :: { 6 : 8. / 14 : x.
59. { 5 : x / 21 : 30 } :: 14 : 16.
60. x : 36 :: { 10 : 14. / 12 : 18.
61. 40 : x :: { 10 : 15. / 10 : 12.

SINGLE RULE OF THREE.

Any proportion, as 3 : 4 :: 6 : 8, may be written

$$\frac{3}{4}=\frac{6}{8}$$

Reducing these fractions to a common denominator, we have

$$\frac{3\times8}{32}=\frac{6\times4}{32}$$

As these fractions are equal, and their denominators the same, their numerators must be equal, or $3\times8=6\times4$. But 3 and 8 are the extremes, and 6 and 4 the means. Hence the following—

PRINCIPLES.

1. The product of the extremes of a proportion is equal to the product of the means.

2. Either extreme equals the product of the means divided by the other extreme.

3. Either mean equals the product of the extremes divided by the other mean.

That is, if any *three terms* of a proportion are given, the remaining term can be found. Hence the name, *Rule of Three.*

To make a statement in proportion is to arrange the three given terms so that two of them form one ratio; the remaining term and the required term another ratio; and the ratios a proportion.

1. If 8 barrels of flour cost \$56, what will 9 barrels cost, at the same rate?

SOLUTION.

bbl. bbl. \$ \$
8 : 9 :: 56 : x

$$x=\frac{56\times9}{8}=\$63$$

The condition, "at the same rate," requires that the 8 barrels of flour have the same ratio to the 9 barrels, as \$56, the *cost* of 8 barrels, is to x dollars, the *cost* of 9 barrels.

2. What will 6 cords of wood cost, if 2 cords cost \$ 7?

3. What will 9 cords of wood cost, if 5 cords cost \$21¼?

4. What will 25 lbs. sugar cost, if 10 lbs. cost 60¢.

5. If 6 hats cost \$24, what will 14 hats cost?

When *more* requires *more*, or *less* requires *less*, the proportion is called *direct*. When *more* requires *less*, or *less* requires *more*, the proportion is called *inverse*. Thus, the more men employed, the more work, and the less men the less work, in a given time; that is, the amount of work varies *directly* as the number of men. But the more men the less time, and the less men the more time to do a given piece of work; that is, the time varies *inversely* as the number of men.

1. If 8 men can build a certain wall in 25 days, how long will it take 12 men to build the same wall?

SOLUTION.

12 : 8 :: 25 : x. In this case the number of days decreases in the same ratio as the number of men increases.

Ans. $16\frac{2}{3}$ d.

2. If 8 hats cost $24, what will 110 hats cost at the same rate?

3. If 2 barrels of flour cost $15, what will 12 barrels cost?

4. If I walk 168 miles in 6 days, how far can I walk at the same rate, in 18 days?

5. If 16 lbs. of sugar cost $1.28, how much will 13 lbs. cost?

6. If 300 barrels of flour cost $2,100, what will 125 barrels cost?

7. If a man travels 638 miles in 22 days, how far will he travel in 50 days?

8. If 27 men can do a piece of work in 15 days, how long will it take 45 men to do it?

9. Find the cost of 4 cords of wood, when 98 cords cost $441.

10. What time should 24 men take to perform a piece of work which 18 men can perform in 15 days?

11. A garrison of 2,100 men has provisions for 9 months, but receives a reinforcement of 600 men. How long will the provisions last?

12. A bankrupt merchant pays 80 cents on a dollar; if he owes A $1,500, how much should A receive?

1. An ocean steamer runs 2,440 miles in 10 days 6 hours. How far will she run in 5 days, 10 hours, at the same speed?

2. The interest on a certain sum of money for 4 mo. 15 da. is $276.75. What is the interest on the same sum, at the same rate, for 6 mo. 18 da.?

3. How high is a steeple whose shadow is 116 feet long, when a flagstaff 45 feet high casts a shadow 60 feet long?

4. If a family of 15 persons has provisions for 8 months, by how many must the family be diminished that the provisions may last 2 years?

5. A garrison of 4,600 men has provisions for 6 months. To what number must the garrison be diminished that the provisions may last 2 years and 6 months?

6. A certain amount of provisions will sustain an army of 9,000 men for 90 days. If the army be increased by 6,000, how long will the same provisions sustain it?

7. If 50 persons consume 600 bushels of wheat in a year, how long would it last 15 persons?

8. A certain work can be done in 120 days by working 6 hours each day. How long would it require to do the same work by working 9 hours a day?

9. If 6 men and 3 boys can do a piece of work in 330 days, how long will it take 9 men and 4 boys to do the same work, if each boy does half as much as a man?

10. If 4 men can do a piece of work in 90 days, how many days will 18 men require to do the same work?

11. If $\frac{5}{8}$ of a barrel of cider costs $\$\frac{6}{7}\frac{9}{7}$, how much will $\frac{7}{8}$ of a barrel cost?

12. If a piece of land 4 rods wide contains $\frac{3}{4}$ of an acre, how many acres would there be if it were $16\frac{2}{3}$ rods wide, and of the same length?

13. If 13 cwt. of iron cost $\$42\frac{1}{4}$, what will 12 cwt. cost?

14. If the earth moves in its orbit about the sun a distance of 597,000,000 miles in 365 days 6 hours, how far on an average does it move each hour?

1. If it costs \$25 for the masonry of 4,000 bricks, how much must be paid for the work which requires 90,000 bricks?

2. If a steamship in 14 days sails a distance of 3,000 miles, in what time, at the same rate, will she sail 24,900 miles?

3. If the diameter of the earth is 8,000 miles, and the loftiest mountain is 5 miles in height, what elevation must be made on a globe 16 in. in diameter to represent accurately the height of such a mountain?

4. If \$100 in 12 months brings an interest of \$7, how much will be the interest of \$100 for 8 months?

5. If the interest of \$100 for 12 months is \$7, what will be the interest of \$75 for the same time and rate?

6. If in 12 months the interest of \$100 is \$7, how long must \$100 be on interest to gain \$10, at the same rate?

7. If a glacier of 60 miles in length move 66 inches per annum, in what time will it move its whole length?

8. If a staff of 10 feet in length makes a shadow 15 feet long, how high is a tree whose shadow measures 90 feet?

9. If a staff 5 feet long casts a shadow 7 feet long, how long is the shadow cast by a steeple 150 feet high?

10. If a steeple 150 feet high casts a shadow 210 feet, what is the height of a shrub which casts a shadow 7 feet long?

11. If 75 kilos of cheese are worth as much as 40 kilos of butter, how many kilos of cheese will pay for 32 kilos of butter?

12. If the interest of \$600 for 6 months is \$15, what sum will gain \$64 in the same time?

13. If an officer's salary amounts to \$6,400 in 4 years, what will it amount to in 15 years?

14. If an officer's salary amounts to \$24,000 in 15 years, in what time will it amount to \$8,800?

15. If 5 men can mow 20 acres of grass in 4 days, by working 6 hours a day, how many days will it take them to mow the same area if they work only 4 hours a day?

16. If $\frac{3}{4}$ of an acre of land is worth \$22.50, what is the value of $25\frac{3}{4}$ acres at the same price?

1. Find the board feet in 5 boards—
   12 feet long, 18 inches wide, $1\frac{1}{2}$ inches thick.
   18 feet long, 14 inches wide, 2 inches thick.
   16 feet long, 15 inches wide, $1\frac{3}{4}$ inches thick.
   15 feet long, 12 inches wide, 2 inches thick.
   10 feet long, 12 inches wide, $1\frac{1}{2}$ inches thick.

2. Find the length of a piece of timber 15 inches wide and 10 inches thick, that contains $8\frac{1}{3}$ cubic feet.

NOTE.—A cubic foot contains 12 board feet.

3. Are the same number of rods required to enclose a square field containing 40 acres as are required to enclose an oblong field containing 40 acres? Prove by diagram.

4. How many acres in a field 84 rods by 20 rods?
5. How many acres in a field 96 rods by 60 rods?
6. How many acres in a field 96 rods by 200 chains?
7. How many acres in a field 215 rods by 50 chains?
8. How many acres in a field 36 rods by 80 rods?
9. How many acres in a field 200 rods by 96 chains?
10. How many acres in a field 70 rods by 96 rods?
11. How many acres in a field 132 rods by 100 chains?

12. Find the cost of carpeting a floor $18\frac{1}{2}$ feet by 16.4 feet, carpet $\frac{3}{4}$ of a yard wide, at $2.25 a yard.

13. Find the cost of carpeting a floor 24 feet 3 inches by 16 feet 4 inches, with carpet $\frac{3}{4}$ of a yard wide, at $1.25 a yard.

14. Find the cost of carpeting a floor 28 feet by 17 feet 9 inches, with carpet $\frac{3}{4}$ of a yard wide, allowing 4 inches for matching figures in each strip except the first, at $2.25 a yard.

15. How many rolls of paper required to cover the walls of a room 24 feet by 30 feet, and 14 feet high above the base board?

16. Find the cost of papering the walls of your schoolroom, at 20 cents a roll.

17. Find the square yards in a floor having the form of a trapezoid, of which the parallel sides are 20 feet 6 inches and 16 feet 4 inches long, and are 8 feet apart.

1. A 90-acre farm in the form of a trapezoid is 80 rods wide. How long is each of the parallel sides, if one is 25% longer than the other?

Find the area of a trapezium* having—

2. A diagonal of 84 in., and two altitudes of 36 in. and 32 in.

3. A diagonal of 50 rds., and two altitudes of 30 rds. and 20 rds.

4. A diagonal of 65 feet and two altitudes of 14 ft. and 18 ft.

5. A diagonal of 34 feet and two altitudes of 16 ft. and 18 ft.

6. A diagonal of 145 in., and two altitudes of 40 in. and 50 in.

7. Changing a circle to triangles, (see page 96) what part of the circle becomes the altitude of the triangles? What part becomes the bases?

8. Write the rule for finding the area of a circle when the radius is given.

9. Write the rule for finding the area of a circle when the circumference is given.

10. If $\frac{5}{6}$ of a bu. of grain costs $\$\frac{7}{8}$, find the cost of $\frac{2}{3}$ of a bu.

11. If $\frac{3}{7}$ of a ship costs \$5,580, find the cost of $\frac{5}{14}$ of it.

12. If $\frac{2}{3}$ of a barrel of flour costs \$3.75, find the cost of 9 bbls.

13. Light travels 192,000 miles a second. If it requires 10 years, of $365\frac{1}{4}$ days each, for light to pass from a certain fixed star to the earth, how far is the star from the earth?

14. Find the time from Washington's birthday to today.

15. Find the time from the landing of the Pilgrims to today.

16. Find the time from the discovery of America by Columbus to today.

17. Find the time from the landing of the Pilgrims to Washington's birthday.

18. Find the time from the discovery of America by Columbus to Washington's birthday.

19. Find the time from the discovery of America by Columbus to the landing of the Pilgrims.

20. Find the cost of 9 hats, if 5 hats cost \$17.50.

* See page 96.

PARTITIVE PROPORTION.

PARTITIVE PROPORTION is the process of separating a number into parts which bear certain relations to each other.

1. A and B together have 2,538 acres of land and B has 5 times as much as A; how many acres has each?

SOLUTION.

B's number = 5 A's,
A's + 5 A's = 6 A's = 2538 acres,
A's = $\frac{1}{6}$ of 2538 acres, or 423 acres,
B's = 5 A's, or 2115 acres.

2. A, B, and C earned $5,740, of which A earned twice as much as C, and B 5 times as much as C. How much did each earn?

3. The value of two farms is $30,784, and one is worth $\frac{5}{8}$ as much as the other. What is the value of each?

SOLUTION.

$\frac{8}{8}$ of the first farm + $\frac{5}{8}$ of it = $\frac{13}{8}$ of it,
$\frac{13}{8}$ of the first farm = $31,784,
$\frac{1}{8} = \frac{1}{13}$ of $30,784 or ——,
$\frac{8}{8} = 8 \times$ ——, or ——,
$\frac{5}{8} = 5 \times$ ——, or ——.

Or,

13 : 8 :: $30,784 : x,
13 : 5 :: $30,784 : x.

4. A and B dug 2,695 rods of ditch, and A dug $\frac{5}{8}$ as much as B. How much did each dig?

5. C and D own 4,498 cows, and C owns $\frac{5}{7}$ as many as D. Required the number owned by each.

6. The sum of two numbers is 7,560, and 4 times the first equals 5 times the second. Find each number.

7. Divide 7,560 in the proportion of 3 and 7.

8. Divide 7,560 in the proportion of $\frac{1}{2}$ and $\frac{2}{3}$.

9. Divide 7,560 in the proportion of $\frac{1}{3}$ and $\frac{1}{4}$.

1. A and B have \$6,000, and 5 times A's money + \$960 equals B's money. How much has each?

SOLUTION.

A's = A's,
B's = 5 A's + \$960,
6 times A's + \$960 = \$6000,
6 times A's = \$5040,
A's = $\frac{1}{6}$ of \$5040, or \$840,
B's = 5 times \$840, + \$960, or \$5160.

2. Two drovers have 7,920 cattle, and the second has $\frac{1}{2}$ as many as the first, plus 750. How many has each?

3. A and B have \$28,126, and A has $\frac{3}{4}$ as many as B, plus \$1,463. How much has each?

4. A farm, a house, and a store cost \$29,058. The farm cost $\frac{16}{21}$ as much as the store, and the house cost $\frac{2}{3}$ as much as the store. What did each cost?

5. Divide the number 16,488 into two parts which are to each other as 17 to 19.

6. A and B together have \$78,318, and A's money is to B's as $\frac{3}{4}$ to $\frac{5}{6}$. How much has each?

NOTE.—The parts are as $\frac{3}{4}$ to $\frac{5}{6}$, or as $\frac{9}{12}$ to $\frac{10}{12}$, or as 9 to 10.

7. Divide the number 37,640 into 3 parts, to each other as 4, 7, and 9.

8. Divide the number 57,648 into 2 parts, to each other as the reciprocals of 5 and 7.

9. Divide the fraction $\frac{3}{4}$ into 2 parts having the same ratio as $\frac{2}{3}$ to $\frac{5}{6}$.

10. Divide 390 into parts having the proportion of 3, 4, and 6.

11. Divide \$7,500 among 3 men, giving to the first \$30 as often as you give \$50 to the second and \$70 to the third.

12. Divide 3,456 into parts having the proportion of 3, 4, and 5.

13. Divide \$405 with two other partners so that one will get \$2 to your \$1 and the other will get \$3 to your \$2.

PARTNERSHIP.

PARTNERSHIP is the association of two or more persons with a joint capital, for the transaction of business.

THE CAPITAL consists of the money or property invested by the partners, and constitutes the *Interest* or *Joint Stock*.

THE RESOURCES OR ASSETS of a firm are the property which it owns and the debts due it.

THE LIABILITIES of a firm are its debts.

THE NET CAPITAL is the excess of the resources over the liabilities.

1. Mr. Jones and Mr. Wilson form a co-partnership, the former putting in $1,250, and the latter, $750. At the end of the year there is a profit of $720. What is the share of each?

SOLUTION.

$1250+$ 750 = $2000.

$2000 : $1250 :: $ 720 : x; or Jones's share = $450.

$2000 : $ 750 :: $ 720 : x; or Wilson's share = $270.

2. A, B, and C engaged in business with a capital of $6,000, of which A invested $1,500, B $2,500 and C $2,000. At the end of a year they had gained $3,000. How should it be divided among them?

NOTE.—Whole investment: A's investment:: whole profits: A's profits.

3. A, B, and C entered into partnership with a capital of $7,500, of which A put in $2,500, B put in $3,000, and C put in the remainder. At the end of the year their gain was $3,000. What was each one's share of it?

4. A and B have a joint stock of $4,200, of which A owns $3,600, and B $600. They gain in one year $2,000. What is each one's share of the profits?

5. A, B, C, and D have $40,000 in trade; each an equal share. At the end of six months their profits amount to $16,000. What is each one's share, allowing A to receive $50 and D $30 out of the profits for extra services?

1. A, B, and C engage in trade. A furnishes \$200, B \$250, and C, \$350. They gain \$100.80. Find each partner's share of the gain.

2. A, B, and C engage in trade. They invest \$1,280, \$1,760, and \$1,920, respectively. Their profits were \$2,790. Find the profit of each.

3. A bankrupt owes three creditors, \$1,750, \$2,100, and \$2,650, respectively. His assets are \$8,450. What should each creditor receive?

4. Two partners gain by trade, \$1,456. One put in \$1,200 capital and the other, \$1,600 capital. What is the gain of each?

5. A freight train was loaded with flour by three merchants. By one was sent 800 barrels, by another, 600 barrels, and by the third, \$1,200 barrels. In a wreck 560 barrels were lost. Find each man's share of the loss.

6. A and B purchase a house for \$2,500, of which A paid \$1,200 and B \$1,300. They receive \$210 rent each year for the same. Find the share of each.

7. A person failing in business finds that all his debts amount to \$4,500, and that he has only \$2,500 to pay them. He owes A \$360 and B \$400. What will each receive?

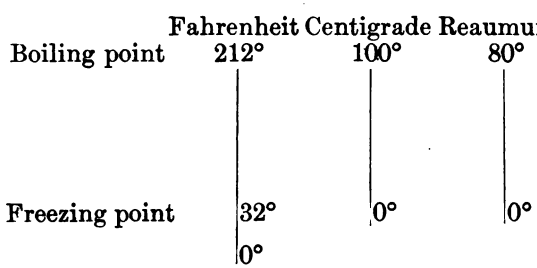
8. Two brothers, one 18 years old and the other 21 years old, contribute in ratio of their ages, \$1,300 to support an aged parent. What did each contribute?

9. Two persons, A and B, hire a pasture for \$30, into which A turned 3 cows and B 5. What part of the \$30 ought each to pay?

10. Five persons, A, B, C, D, and E, share \$2,400. A receives $\frac{1}{6}$ of it, B, $\frac{1}{4}$ of it, C, $\frac{3}{8}$ of it, and D and E are to divide the remainder in proportion to the numbers 3 and 7. How much does each one receive?

11. A and B form a partnership with a capital of \$6,000, of which A puts in \$3,500 and B \$2,500. They gain \$1,500. What is the gain of each?

12. Make an original problem in partnership.



F. = Fahrenheit; C. = Centigrade; R. = Reaumur.

Temperature below 0° is indicated by a minus sign (—).

212°—32°=180°, the difference, F. scale, between freezing and boiling.

1. Memorize—

180° F. = 100° C. = 80° R.
90° F. = 50° C. = 40° R.
45° F. = 25° C. = 20° R.
18° F. = 10° C. = 8° R.
9° F. = 5° C. = 4° R.

2. Express 41° F. in the C. scale; in the R. scale.
3. Express 5° C. in the F. scale; in the R. scale.
4. Express 12° R. in the F. scale; in the C. scale.
5. Express 57° F. in the C. scale; in the R. scale.
6. Express 23° F. in the C. scale; in the R. scale.
7. Express —10° C. in the F. scale; in the R. scale.
8. Express —12° R. in the F. scale; in the C. scale.
9. Express 50° F. in the C. scale; in the R. scale.
10. Express 30° C. in the F. scale; in the R. scale.
11. Express 16° R. in the F. scale; in the C. scale.
12. Express 59° F. in the C. scale; in the R. scale.
13. Express —16° R. in the F. scale; in the C. scale.
14. Express —15° C. in the F. scale; in the R. scale.
15. Express 185° F. in the C. scale; in the R. scale.
16. Express 36° R. in the F. scale; in the C. scale.
17. Express 55° C. in the F. scale; in the R. scale.
18. Express 167° F. in the C. scale; in the R. scale.

1. Memorize—

2 times $6\frac{1}{4}\% = 12\frac{1}{2}\%$.	10 times $6\frac{1}{4}\% = 62\frac{1}{2}\%$.
3 times $6\frac{1}{4}\% = 18\frac{3}{4}\%$.	11 times $6\frac{1}{4}\% = 68\frac{3}{4}\%$.
4 times $6\frac{1}{4}\% = 25\ \%$.	12 times $6\frac{1}{4}\% = 75\ \%$.
5 times $6\frac{1}{4}\% = 31\frac{1}{4}\%$.	13 times $6\frac{1}{4}\% = 81\frac{1}{4}\%$.
6 times $6\frac{1}{4}\% = 37\frac{1}{2}\%$.	14 times $6\frac{1}{4}\% = 87\frac{1}{2}\%$.
7 times $6\frac{1}{4}\% = 43\frac{3}{4}\%$.	15 times $6\frac{1}{4}\% = 93\frac{3}{4}\%$.
8 times $6\frac{1}{4}\% = 50\ \%$.	16 times $6\frac{1}{4}\% = 100\ \%$.
9 times $6\frac{1}{4}\% = 56\frac{1}{4}\%$.	

2. How find $12\frac{1}{2}\%$ of a number, if $6\frac{1}{4}\%$ is given?

3. How find $6\frac{1}{4}\%$ of a number, if $12\frac{1}{2}\%$ is given?

4. How find 25 % of a number, if $6\frac{1}{4}\%$ is given?

5. How find 25 % of a number, if $12\frac{1}{2}\%$ is given?

6. How find $18\frac{3}{4}\%$ of a number, if $6\frac{1}{4}\%$ is given?

7. How find $18\frac{3}{4}\%$ of a number, if $12\frac{1}{2}\%$ is given?

8. How find $18\frac{3}{4}\%$ of a number, if 25 % is given?

9. How find $37\frac{1}{2}\%$ of a number, if $6\frac{1}{4}\%$ is given?

10. How find $37\frac{1}{2}\%$ of a number, if $12\frac{1}{2}\%$ is given?

11. How find $37\frac{1}{2}\%$ of a number, if 25 % is given?

12. What is $6\frac{1}{4}\%$ of a number, if $62\frac{1}{2}\%$ of it is 48?

13. What is $12\frac{1}{2}\%$ of a number, if $62\frac{1}{2}\%$ of it is 120?

14. What is $37\frac{1}{2}\%$ of a number, if $62\frac{1}{2}\%$ of it is 250?

15. What is $68\frac{3}{4}\%$ of a number, if $62\frac{1}{2}\%$ of it is 240?

16. What is 75 % of a number, if $62\frac{1}{2}\%$ of it is 1500?

17. What is $62\frac{1}{2}\%$ of a number, if 75 % of it is 1300?

18. What is $43\frac{3}{4}\%$ of a number, if 50 % of it is 1200?

19. What is $81\frac{1}{4}\%$ of a number, if 75 % of it is 3000?

20. If 30 books cost \$50, how many books of the same kind will \$56.25 buy?

21. If \$37.50 pays for 12 acres of land, how many acres at the same price can be bought for \$31.25?

22. If \$7,500 pays for 240 acres of land, how many acres at the same price can be bought for \$6,875?

NOTE.—**The relation is seen by dividing the amounts by 100.**

23. If $43\frac{3}{4}\%$ of a number is 42, what is 50%?

1. Memorize—

2 times $8\frac{1}{3}\% = 16\frac{2}{3}\%$.	8 times $8\frac{1}{3}\% = 66\frac{2}{3}\%$.
3 times $8\frac{1}{3}\% = 25\ \%$.	9 times $8\frac{1}{3}\% = 75\ \%$.
4 times $8\frac{1}{3}\% = 33\frac{1}{3}\%$.	10 times $8\frac{1}{3}\% = 83\frac{1}{3}\%$.
5 times $8\frac{1}{3}\% = 41\frac{2}{3}\%$.	11 times $8\frac{1}{3}\% = 91\frac{2}{3}\%$.
6 times $8\frac{1}{3}\% = 50\ \%$.	12 times $8\frac{1}{3}\% = 100\ \%$.
7 times $8\frac{1}{3}\% = 58\frac{1}{3}\%$.	

2. How find $16\frac{2}{3}\%$ of a number, if $8\frac{1}{3}\%$ is given?

3. How find $8\frac{1}{3}\%$ of a number, if $16\frac{2}{3}\%$ is given?

4. How find 25 % of a number, if $8\frac{1}{3}\%$ is given?

5. How find $8\frac{1}{3}\%$ of a number, if 25 % is given?

6. How find 25 % of a number, if $16\frac{2}{3}\%$ is given?

7. How find $16\frac{2}{3}\%$ of a number, if 25 % is given?

8. How find $16\frac{2}{3}\%$ of a number, if 50 % is given?

9. How find $33\frac{1}{3}\%$ of a number, if 50 % is given?

10. How find 25 % of a number, if $33\frac{1}{3}\%$ is given?

11. How find $66\frac{2}{3}\%$ of a number, if 25 % is given?

12. What is $8\frac{1}{3}\%$ of a number, if $83\frac{1}{3}\%$ of it is 100?

13. What is $16\frac{2}{3}\%$ of a number, if $83\frac{1}{3}\%$ of it is 120?

14. What is 25 % of a number, if $83\frac{1}{3}\%$ of it is 240?

15. What is 25 % of a number, if $33\frac{1}{3}\%$ of it is 400?

16. What is 50 % of a number, if $66\frac{2}{3}\%$ of it is 800?

17. What is 50 % of a number, if $33\frac{1}{3}\%$ of it is 800?

18. What is 75 % of a number, if $66\frac{2}{3}\%$ of it is 800?

19. What is $66\frac{2}{3}\%$ of a number, if 75 % of it is 450?

20. If 8 chairs cost $\$66\frac{2}{3}$, how many chairs of the same kind will \$75 buy?

21. If 84 acres of land cost \$5,000, how many acres at the same price will $\$6{,}666\frac{2}{3}$ buy?

22. If 100 acres of land cost $\$3{,}333\frac{1}{3}$, how many acres at the same price will \$5,000 buy?

23. If $\frac{1}{4}$ of an acre is worth \$30, what is $8\frac{1}{3}\%$ of an acre worth, at the same rate?

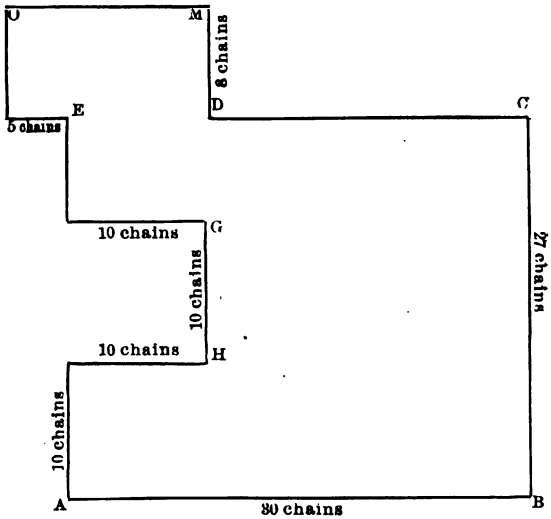
24. Count by $8\frac{1}{3}$ from $8\frac{1}{3}$ to 100, and back from 100 to $8\frac{1}{3}$.

25. If $83\frac{1}{3}\%$ of a number is 50, what is $91\frac{2}{3}\%$?

1. Memorize—

2 times $12\frac{1}{2}\% = 25\ \%$.	8 times $12\frac{1}{2}\% = 100\ \%$.
3 times $12\frac{1}{2}\% = 37\frac{1}{2}\%$.	2 times $16\frac{2}{3}\% = 33\frac{1}{3}\%$.
4 times $12\frac{1}{2}\% = 50\ \%$.	3 times $16\frac{2}{3}\% = 50\ \%$.
5 times $12\frac{1}{2}\% = 62\frac{1}{2}\%$.	4 times $16\frac{2}{3}\% = 66\frac{2}{3}\%$.
6 times $12\frac{1}{2}\% = 75\ \%$.	5 times $16\frac{2}{3}\% = 83\frac{1}{3}\%$.
7 times $12\frac{1}{2}\% = 87\frac{1}{2}\%$.	6 times $16\frac{2}{3}\% = 100\ \%$.

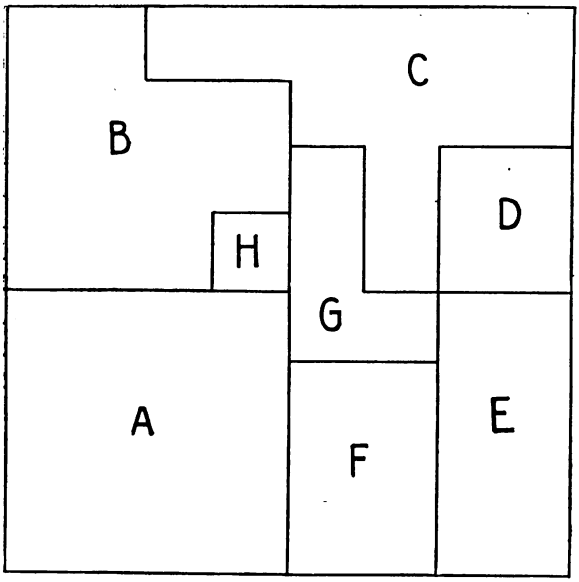
2. How find $12\frac{1}{2}\%$ of a number, if $37\frac{1}{2}\%$ is given?
3. How find $25\ \%$ of a number, if $37\frac{1}{2}\%$ is given?
4. How find $50\ \%$ of a number, if $37\frac{1}{2}\%$ is given?
5. How find $37\frac{1}{2}\%$ of a number, if $25\ \%$ is given?
6. How find $37\frac{1}{2}\%$ of a number, if $50\ \%$ is given?
7. How find $62\frac{1}{2}\%$ of a number, if $50\ \%$ is given?
8. How find $62\frac{1}{2}\%$ of a number, if $75\ \%$ is given?
9. How find $75\ \%$ of a number, if $62\frac{1}{2}\%$ is given?
10. How find $50\ \%$ of a number, if $62\frac{1}{2}\%$ is given?
11. How find $87\frac{1}{2}\%$ of a number, if $75\ \%$ is given?
12. How find $12\frac{1}{2}\%$ of a number, if $62\frac{1}{2}\%$ is given?
13. How find $37\frac{1}{2}\%$ of a number, if $62\frac{1}{2}\%$ is given?
14. How find $62\frac{1}{2}\%$ of a number, if $37\frac{1}{2}\%$ is given?
15. How find $87\frac{1}{2}\%$ of a number, if $37\frac{1}{2}\%$ is given?
16. How find $37\frac{1}{2}\%$ of a number, if $87\frac{1}{2}\%$ is given?
17. What is $33\frac{1}{3}\%$ of a number, if $16\frac{2}{3}\%$ is 100?
18. What is $33\frac{1}{3}\%$ of a number, if $50\ \%$ is 120?
19. What is $50\ \%$ of a number, if $33\frac{1}{3}\%$ is 100?
20. What is $66\frac{2}{3}\%$ of a number, if $50\ \%$ is 150?
21. What is $50\ \%$ of a number, if $66\frac{2}{3}\%$ is 100?
22. What is $100\ \%$ of a number, if $83\frac{1}{3}\%$ is 100?
23. What is $83\frac{1}{3}\%$ of a number, if $100\ \%$ is 120?
24. Find $25\ \%$ of a number, if $37\frac{1}{2}\%$ is 120.
25. Find $50\ \%$ of a number, if $37\frac{1}{2}\%$ is 120.
26. Find $50\ \%$ of a number, if $62\frac{1}{2}\%$ is 120.
27. Find $62\frac{1}{2}\%$ of a number, if $50\ \%$ is 120.
28. Find $62\frac{1}{2}\%$ of a number, if $75\ \%$ is 120.
29. Find $87\frac{1}{2}\%$ of a number, if $75\ \%$ is 120.



1. Express in acres the area of the figure.
2. Draw a line from D to E, and find the area of each part.
3. Draw a line from D to F, and find the area of each part.
4. Draw a line from D to G, and find the area of each part.
5. Draw a line from C to F, and find the area of each part.
6. Draw a line from C to G, and find the area of each part.
7. Draw a line from C to H, and find the area of each part.
8. Draw a line from B to D, and find the area of each part.
9. Draw a line from B to G, and find the area of each part.
10. Draw a line from B to H, and find the area of each part.
11. Draw a line from B to O, and find the area of each part.

NOTE.—Extend the lines BC and OM, find the area of the entire triangle, and subtract the area cut out by the lines DM and DC.

12. Extend the line FG to the line BC, and find the area of each part.
13. How many rods around the entire area?
14. Draw the lines EM and DF, and find the area of the figure enclosed.



Quarter Section.

1. The above figure represents a quarter section of land. Find the value of H at $40 an acre.

2. Compare the areas of G and D. Compare the rods of fencing required to enclose them.

3. Which piece requires the most fence to enclose it?

4. What will C and E yield at $3 per acre for pasturage?

5. F and B yielded 35 bushels of wheat per acre. What was the value of the crop at 60 cents a bushel?

6. What per cent of the whole quarter section is A? Is D? Is H? Is G? Is E? Is F?

7. If a road 4 rods wide is established through the center of the quarter section, running north and south, how much is taken from each piece cut by the road?

8. How many rods of fencing in the entire quarter, reckoning the road fenced on both sides?

SHORT METHODS.

Annexing one cipher to an integer multiplies by 10; annexing two ciphers, multiplies by 100, etc. Hence, to multiply a number—

By $2\frac{1}{2}$, annex one cipher and divide by 4. Why?
By $3\frac{1}{3}$, annex one cipher and divide by 3. Why?
By 25 , annex two ciphers and divide by 4. Why?
By $33\frac{1}{3}$, annex two ciphers and divide by 3. Why?
By $16\frac{2}{3}$, annex two ciphers and divide by 6. Why?
By $12\frac{1}{2}$, annex two ciphers and divide by 8. Why?
By $12\frac{1}{2}$, multiply by 10 and add $\frac{1}{4}$ of the product.
By $13\frac{1}{3}$, multiply by 10 and add $\frac{1}{3}$ of the product.
By 15 , multiply by 10 and add $\frac{1}{2}$ of the product.
By 125 , multiply by 100 and add $\frac{1}{4}$ of the product.
By 1250 , multiply by 1000 and add $\frac{1}{4}$ of the product.

1. $246\times 2\frac{1}{2}=$
2. $246\times 3\frac{1}{3}=$
3. $246\times 25=$
4. $246\times 33\frac{1}{3}=$
5. $624\times 16\frac{2}{3}=$
6. $624\times 12\frac{1}{2}=$
7. $96\times 13\frac{1}{3}=$
8. $84\times 125=$
9. $84\times 1250=$
10. $756\times 15=$
11. $756\times 112\frac{1}{2}=$
12. $747\times 3\frac{1}{3}=$
13. $747\times 13\frac{1}{3}=$
14. $747\times 133\frac{1}{3}=$
15. $576\times 112\frac{1}{2}=$
16. $576\times 116\frac{2}{3}=$
17. $576\times 125=$
18. $576\times 133\frac{1}{3}=$

19. Find the cost of 24 hats @ $\$2.12\frac{1}{2}$ each.
20. Find the cost of 360 volumes @ \$1.25 a volume.
21. Find the cost of 24 town lots @ \$ 125 per lot.
22. Find the cost of 24 town lots @ \$1125 per lot.
23. Find the cost of 24 town lots @ \$1250 per lot.
24. Find the cost of 660 lbs. of fruit @ $\$ .16\frac{2}{3}$ per pound.
25. Find the cost of 650 tons of coal @ $\$3.33\frac{1}{3}$ per ton.
26. Find the cost of 1250 ft. lumber @ \$16.40 per M.
27. Find the cost of 240 tons of hay @ \$12.50 per ton.
28. Find the cost of 324 tons of hay @ $\$13.33\frac{1}{3}$ per ton.
29. Find the cost of $112\frac{1}{2}$ tons of hay @ \$9.60 per ton.

1. If 2 desks cost \$45, what will 3 cost at the same rate?
   Suggestion: One desk cost \$——; three will cost \$——.
2. If 20 hats cost \$80, what will be the cost of 25 hats?
   Suggestion: 5 hats cost \$——; 25 hats cost \$——.
3. If 15 hats cost \$45, what will be the cost of 25 hats?
4. If 8 hats cost \$32, what will be the cost of 12 hats?
5. If 40 yds. of cloth cost \$80, what will be the cost of 25 yds.?
6. If 32 yds. of cloth cost \$160, what will be the cost of $8\frac{3}{4}$ yds.?
7. If $3\frac{1}{3}$ yds. of cloth cost \$2.40, what will be the cost of 30 yds.?
8. If 10 yds. of cloth cost \$7.20, what will be the cost of 35 yds.?
9. If $8\frac{1}{3}$ yds. of cloth cost \$16, what will be the cost of 25 yds.?
10. If $12\frac{1}{2}$ yds. of cloth cost \$8, what will be the cost of $112\frac{1}{2}$ yds.?

Find the cost—

11. Of a house and lot, if $\frac{3}{5}$ the cost was \$1,800.
12. Of a house and lot, if $\frac{5}{8}$ the cost was \$4,000.
13. Of a house and lot, if $\frac{8}{9}$ the cost was \$8,400.
14. Of a house and lot, if $\frac{4}{7}$ the cost was \$5,600.
15. Of a house and lot, if \$4,800 is 20 % more than the cost.
16. Of a house and lot, if \$4,500 is 25 % more than the cost.
17. Of a house and lot, if \$4,500 is 25 % less than the cost.
18. Of a house and lot, if \$3,600 is $12\frac{1}{2}$% more than the cost.
19. Of a house and lot, if \$3,600 is $12\frac{1}{2}$% less than the cost.
20. Of a house and lot, if \$3,600 is 40 % less than the cost.
21. Of a lot that gains $\frac{1}{5}$ the cost when sold for \$2,400.
22. Of a lot that gains $\frac{1}{4}$ the cost when sold for \$2,400.
23. Of a lot that gains $\frac{1}{3}$ the cost when sold for \$2,400.
24. Of a lot that gains $\frac{1}{8}$ the cost when sold for \$3,600.
25. Of a lot that loses $\frac{1}{8}$ the cost when sold for \$3,500.
26. Of a lot that loses $\frac{1}{9}$ the cost when sold for \$3,600.
27. Of a lot that loses $\frac{1}{6}$ the cost when sold for \$3,600.
28. Of a lot that loses $\frac{2}{5}$ the cost when sold for \$2,400.
29. Of a lot that gains $\frac{2}{5}$ the cost when sold for \$2,800.
30. Of a lot that gains $\frac{3}{7}$ the cost when sold for \$2,800.
31. Of a lot that loses $\frac{3}{7}$ the cost when sold for \$2,800.
32. Of a lot that gains $\frac{5}{8}$ the cost when sold for \$2,800.

1. Find the weight of 9 loads of hay, each weighing 1 ton 350 lbs.

2. A 6-acre field, rectangular in form, is 99 feet wide. Find the distance around it.

3. From an acre lot is fenced off one piece 7 rods square and another piece containing 7 square rods. What part of the acre is still unfenced?

4. Find the amount due a wholesale house for the following goods bought by a country merchant:

24 yards of silk	@	$1.16⅔ ℔ yd.	.....
64 yards of calico	@	.43¾ ℔ yd.	.....
165 yards of cloth	@	1.33⅓ ℔ yd.	.....
120 yards of cloth	@	1.08⅓ ℔ yd.	.....
24 gallons of wine	@	1.87½ ℔ gal.	.....
24 barrels of flour	@	4.41⅔ ℔ bbl.	.....
		Total amount,	.....

What decimal part of—

5. A year are 9½ months?
6. A day are 9½ hours?
7. A mile are 72 yards?
8. A ton are 64 pounds?
9. A pound are 7 oz. 19 pwt.
10. A £ are 2.4 shillings?
11. An acre are 24 sq. yds. 1.8 sq. ft.
12. A ream of paper are 9 sheets?
13. A gallon are .25 pints?
14. A cwt. are .5 pounds?

Reduce to lower denominations—

15. .375 of a yard.
16. .348 of a lb. apoth. wght.
17. .1875 of a ream of paper.
18. .055 of a ton.
19. .025 of a shilling.
20. .3375 of an acre.

Reduce—

21. $\frac{3}{5}$ of a grain troy to lbs.
22. $\frac{5}{6}$ ton to pounds.
23. $\frac{4}{5}$ ℈ to pounds.
24. $\frac{7}{8}$ of a week to minutes.
25. $\frac{3}{5}$ of a pint to bushels.
26. $\frac{9}{16}$ of a lb. troy to grains.
27. $\frac{4}{7}$ oz. avdp. to cwt.
28. $\frac{2}{3}$ of an inch to yard.
29. $\frac{3}{160}$ bushel to pint.
30. $1\frac{3}{5}$ pecks to bushel.
31. $\frac{1}{5}$ of a cwt. to ton.
32. $\frac{1}{8}$ of a gill to gallon.

33. Make an original problem in denominate numbers.

1. Use aliquot parts in finding the amount of the following bill:

4 bu. 3 pks. 5 qts. seed	@	$4.00 ℔ bushel.
10 bu. 3 pks. 7 qts. 1 pt	@	3.20 ℔ bushel.
10 bu. 1 pk. 1 qt. 1 pt	@	2.56 ℔ bushel.
3 lbs. 14 oz. spices	@	.24 ℔ pound.
4 lbs. 12 oz. butter	@	.30 ℔ pound.
	Total	

2. Reduce 2 oz. 2 pwt. 2 grains to the fraction of a pound troy.

3. Reduce 2 cord ft. 8 cu. ft. to the fraction of a cord.

4. Reduce 1 gal. 1 qt. 1 pt. 1 gi. to the fraction of a barrel.

5. Find the sum of $\frac{1}{4}$ of a yard, $\frac{1}{4}$ of a foot, and $\frac{1}{4}$ of an in.

6. What part of 3 rods are 4 yards 1.5 feet?

7. Find the sum of $\frac{1}{8}$ of a week .75 of a day and $\frac{2}{3}$ of an hour.

8. Find the sum of $\frac{3}{20}$ of a common year, $\frac{5}{12}$ of a day and $\frac{2}{3}$ of 19.5 hours.

9. Three loads of hay weighed 1.2 tons, $1\frac{1}{2}$ tons, and $18\frac{1}{2}$ cwt. Find the weight of the three loads together.

10. A section of land is divided into lots each containing $\frac{3}{5}$ of an acre. How many lots are in the section?

11. A section of land is divided into 15 equal farms. Find the exact size of each farm.

12. A pile of wood containing 45 cords is 4 feet wide and 8 feet high. How long is the pile?

13. Divide 130 lbs. 10 oz. 16 pwt. 20 grs. by 154.

14. Divide 6 fur. 1 rd. 2.5 yds. 10 in. by 5 and by 8, and find the difference between the quotients.

15. How many angles of 3° 45′ will fill the space of a semicircle?

Multiply by the numerator and divide the product by the denominator to find—

16. $\frac{2}{3}$ of $4\frac{1}{2}$ tons.

17. $\frac{5}{6}$ of 3 bu. 3 pks. 3 qts.

18. $\frac{1}{8}$ of 1 yr. 5 mo. 20 da.

19. $\frac{3}{8}$ of 3 mi. 100 rds. 12 ft.

20. Make a bill of goods bought at a clothing store.

1. What fraction of 2 pounds avdp. is $\frac{5}{8}$ of an oz.?

2. What fraction of 2 pounds troy is $\frac{5}{8}$ of an oz.?

3. Find the sum of £$\frac{3}{5}$ and $\frac{7}{8}$ of a shilling.

4. Find the sum of $\frac{5}{8}$ of a mile $\frac{3}{8}$ of a rod and .75 of a yard.

5. A quarto book required 2 reams 10 quires and 10 sheets of paper. How many pages did it have?

6. In running 40 miles how many times will a bicycle wheel turn if it is 15.5 feet in circumference?

7. A can do a piece of work in 3 days, B in 4 days, and C in 6 days. In what time can they do it when working together?

8. What was the cost of grading a country road for a distance of 8 miles 200 rods at $11.75 a rod?

9. If 5 men can build a mile of fence in $7\frac{1}{2}$ days, how many men will be required to build it in $1\frac{1}{4}$ days?

10. Find the total cost—

Of 2125 pounds of coal at $3.00 a ton.
Of 1225 pounds of coal at 4.00 a ton.
Of 2550 pounds of coal at 3.50 a ton.
Of 3330 pounds of coal at 3.75 a ton.

11. A milkman bought 80 quarts of milk for $4.00 and lost, by an accident, 8 quarts. At what price a quart must he sell the remainder to make a profit of 4 cents a quart on the whole?

12. A gentleman started at 10:30 Monday forenoon on a journey, and found that by Wednesday at 11:20 P. M. he had traveled 1,000 miles. What was the average rate per hour?

13. The base of a triangle is 36.25 inches, and the area is $894\frac{1}{2}$ square inches. Find the altitude.

14. Divide $1,471.84 among 4 men in the ratio of 2, 3, 4, and 7.

15. A ditch is dug around a lot 240 by 164 feet. It is 3.5 feet wide and 6.75 feet deep. Find the cubic contents of the ditch.

16. A has $9,600, and $\frac{5}{8}$ of his money plus $1,200 equals $\frac{7}{8}$ of B's. Find B's money.

17. Make an original problem in simple proportion.

1. Find the area of a circle having a circumference of 628.32 inches.

2. Find the cost of laying a floor 36.25 feet long and 16.5 feet wide at $3.50 a square?

3. Find the cost of the shingles for a roof 40 feet long, the rafters on each side 18 feet 8 inches long, at $3.60 a thousand.

4. A grocer's weighing scales are not justly balanced, so that a pound appears to weigh but 14 ounces. What will 240$\frac{3}{8}$ pounds appear to weigh by the false balances?

5. In a certain fort were sufficient provisions to last 2,000 soldiers 10 months. How many more should come so that the provisions would last but 4 months?

6. A garden is 12 rods square. A walk 6 feet wide is made around the garden, just inside the fence. Find the cost of the walk at $.75 a square yard.

7. A room 30 feet long requires 120 yards of carpet $\frac{3}{4}$ of a yard wide. How wide is the room?

8. A lot 222 feet long and 120 feet wide is surrounded by a tight board fence 6 feet high. Find the cost of the boards required at $6 per M., and of painting both sides of the fence at 6 cents a square yard.

9. How many common bricks 8x4x2 inches are required to make a pile 15 feet 6 inches long, 12 feet 4 inches wide and 8 feet high?

10. In a schoolroom 30 feet long, 24 feet wide, and 12 feet high, are 40 pupils, each breathing 10 cubic feet of air in a minute. In what time will all the pupils breathe as much air as the room contains?

11. How many gallons of air in a room 21.5 feet long, 15 feet wide and 10 feet high?

12. Four persons divided $3,838 in proportion of $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{1}{2}$. Find each person's share.

13. A rectangular field containing 8 acres is 330 feet wide. Find its length.

14. Make an original problem in land measure.

1. Draw a diagram of the four farms and find the cost of fencing them at 50 cents a rod.

A section of land is divided into four farms of 160 acres each, as follows:

(1) N. ½ of the NW., the SW. of the NW. and the NW. of the SW.

(2) E. ½ of the NE., the NW. of the NE. and the NE. of the SE.

(3) S. ½ of the SE., the NW. of the SE. and the SE. of the NE.

(4) E. ½ of the SW., the SW. of the SW. and the SE. of the NW.

2. One watch gains 4½ minutes an hour and another loses 2½ minutes an hour. If set right at noon Friday, what time will each show at noon the next Friday, and what will be the difference between the hours they indicate?

3. A man gave $3,600 to three sons. To the second he gave $400 more than to the first, and to the third he gave $700 more than to the second. How much did each receive?

4. Find the area of the largest square that can be drawn in a circle 20 inches in diameter.

NOTE.—Think of the diameter as the diagonal of the square, and observe the triangles.

5. A cow is tied to a stake in a meadow by a rope 24.5 feet in length. Over what area can she graze?

6. Find the area of a public road, 60 feet wide, around a quarter section of land, ½ the width being taken from the quarter section.

7. Estimate the square yards of plastering in a room 15 feet long, 12 feet wide, and 9½ feet high, deducting for a base board 1 foot wide around the room and for ½ the space of a door 3½ feet by 9 feet, and ½ the space of 3 windows 3 feet by 6½ feet.

8. A walk 6 feet wide extends from the schoolhouse to the street, 100 feet. How many common bricks 4x2x8 inches are required to pave it?

1. At \$10.50 per M., find the cost—

Of 20 pieces 16 feet long, 2x4 inches.

Of 10 pieces 14 feet long, 2x6 inches.

Of 300 planks 16 feet long, 15 inches wide, $1\frac{1}{2}$ inches thick.

Of 50 planks 15 feet long, 16 inches wide, $2\frac{1}{2}$ inches thick.

Of 75 planks 12 feet long, 10 inches wide, 2 inches thick.

2. A man owned a piece of land 40 rods by 28 rods. He sold $12\frac{1}{2}$ per cent of it at the rate of \$160 an acre. What did he receive for the part sold?

3. A room is 15 feet wide and 16 feet 8 inches long. Find the cost of covering the floor with matting 1 yard wide at \$ $.37\frac{1}{2}$ a yard?

4. A farmer owns the S. $\frac{1}{2}$ of the SW. $\frac{1}{4}$ of a section. A public road 4 rods wide runs along the south and west sides, $\frac{1}{2}$ of the width being taken from the farm. How many acres remaining in the farm?

5. A piece of land is 30 chains long. Find its width in rods if it cost \$325 at \$25 an acre.

6. Three men formed a partnership, in which one contributed \$500, another \$700, and a third a stock of goods. They gained \$1,728, and the share of the third man was \$768. Find the value of the stock of goods put into the business.

7. At the rate of \$1,000 per acre, find the value of a lot 16 rods by 38 rods.

8. If 20 men can mow a field in 15 days, how many men must be added so that the field can be mowed in 12 days?

9. A rectangular field, 40 rods by 24 rods, is surrounded by a fence 4 boards high. The boards are 16 feet long, the posts are placed 8 feet apart, except the short panels at the corners. The posts cost 8 cents each, and the boards cost \$12 per M. Find the total cost of the lumber.

10. A shed is 10 feet high, 12 feet wide and 16 feet long. What will it cost to fill it with wood at \$4.75 a cord?

11. Find the weight of water required to fill a tank 4 feet long, $2\frac{1}{2}$ feet deep, and 3 feet wide.

1. Find the latitude of 745 miles north of the equator.

2. Find the weight avoirdupois of 200 lbs. troy weight.

3. If a man breathes at the rate of 15 times in a minute, how many times will he breathe from 6:00 A. M. Monday to 1:15 P. M. Wednesday of the same week?

4. If a boy spends 40 minutes each day in reading good literature, how much time will he spend in reading during a period of 15 years?

5. Telegraph poles were 60 yards apart, and a passenger counted 35 poles in $1\frac{1}{4}$ minutes. What was the speed of the train per hour?

6. If 20 reapers can cut a field of grain in 6 days, in how many days will 30 reapers do it? How many reapers required to cut it in 5 days?

7. A, B, and C formed a partnership in business. A put in \$8,000, B \$4,500 and C \$3,500, and their gains for the first year were \$3,200. What was each man's share of the profit?

8. Three men enter into partnership. A puts in \$2,160, B twice as much as A, and C twice as much as the difference between the amounts that A and B put in. Find each man's share of a profit of \$500?

9. Three partners had a gain of \$6,250 to divide according to each member's investment. A invested \$10,000, B \$15,000, and C \$25,000. What was the gain of each?

10. A piece of work can be done in 40 days by 25 men. After 18 days, 13 men quit work. In how many more days can the rest finish the work?

11. A rectangular enclosure measures 24 rods by 15. Next to the fence, extending all around, there is a walk $7\frac{1}{2}$ feet wide. The rest of enclosure is under cultivation. What is the area of the cultivated portion?

12. Brown and Jones trade in company and gained \$120. Brown's share of the capital was \$1,000, and Jones's share of the gain was \$70. What was Jones's share of the capital?

1. Multiply 1.25 of .8¾ by .8 of $\frac{\frac{7}{8}}{2}$.

2. Multiply $\frac{.5}{.21}$, $\frac{.6}{14}$, $\frac{3.5}{.06}$ and 4.2.

3. Divide 1,043 into three parts having a ratio of ⅝, ⅔, and ½.

4. Divide $343 among 4 men, so that B shall have twice as much as A, C shall have as much as A and B, and D shall have as much as B and C.

5. Find the cost of 2-inch plank 16 feet long for a walk 110 feet long and 4 feet wide, at $18.75 per M., board measure.

6. Find the cost of 2-inch plank 12 feet long for a walk 50 feet long, 6 feet wide, at $14.25 per M., board measure.

7. From a 20-gallon can 8 gals. 3 qts. 1 pt. were drawn. What is the remainder worth at $.16 per gallon?

8. A lot 35 feet wide and 140 feet deep has a sidewalk laid along one side and across one end. The walk is 4 feet wide, made of 2-inch plank laid crosswise on two stringers of 4x4 scantling. What is the cost of the lumber at $24 per M.

9. How many barrels of flour can be bought for $104½, at the rate of ⅜ of a barrel for $6⅝?

10. At 35 cents a roll, what will be the cost of papering the walls of a room 18 feet long, 14 feet wide and 12 feet high, making no allowance for openings, the base board being 9 inches high?

11. A carpenter working alone can build a house in 28 days. With the help of another man he can build it in 18 days. In how many days can the helper alone build the house?

12. How many more revolutions will the fore wheels of a carriage make in one mile than the hind wheels, their diameters being 4 feet and 4 feet 4 inches, respectively?

13. A horse is lariated to a stake in an open field by a rope 60 feet in length. Over how many square rods can he graze?

14. A and B can do ¼ of a piece of work in 1 day; A can do ⅛ of it in 1 day. How much can B do in 1 day? In how many days can A do the whole work? In how many days can B do the whole work?

1. Divide 440 into three parts having the ratio of 3, $3\frac{1}{3}$, and 7.

2. Divide \$1,410 among A, B, and C in ratio of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$.

3. Find the least number that will give 4 as a remainder when divided by 6, 8, or 9.

4. If the quotient is 70, the remainder 26, and the dividend 2,896, what is the divisor?

5. A, B, and C agree to contribute \$365 towards building a church, which is to be at the distance of 2 miles from A, $2\frac{7}{8}$ miles from B, and $3\frac{1}{2}$ miles from C. They agree that their shares shall be proportional to the reciprocals of their distances from the church. What ought each to contribute?

6. Find the length of the shortest line that can be exactly measured by a yard measure, a 10-foot pole, and $\frac{1}{2}$ a chain.

7. My bedroom contains 1,344 cubic feet. It is 14 feet long and 12 feet wide. How high is it?

8. A fence 5 boards high is built around a square field containing 10 acres; the top board is 4 inches wide, the base board is 10 inches wide, the middle boards each 6 inches wide. What is the cost of the lumber at \$12.50 per M.?

9. If a field 40 rods square yields 50 bushels of corn to the acre, what is the crop worth at $33\frac{1}{3}$¢ per bushel? How many posts set 8 feet apart will be required to fence it? How many boards $16\frac{1}{2}$ feet long are required to put up a 5-board fence around it? A strip 2 rods wide is given for a road from two sides of this field; how many square rods are given to the public?

10. Mark off on a large sheet of paper, beginning about 6 inches to the right of the lower left-hand corner, 5 inches to the right, thence 5 inches up, thence 10 inches to the right, thence 10 inches up, 10 inches to the left, thence 5 inches up, thence 5 inches to the left, thence 5 inches down, thence 5 inches to the left, thence 10 inches down, thence 5 inches to the right, thence to the place of beginning.

(1) Find the area of the figure. (2) Cut and fold the figure into the form of a solid, and find the contents.

1. A city lot 50 by 100 feet has a building on it 25 by 40 feet. What part of the lot is covered by the building?

2. Begin at a point near the lower left-hand corner of your paper and draw a horizontal line to represent 15 rods, thence up 10 rods, thence to the left 5 rods, thence up 10 rods, thence to the left 5 rods, thence down 15 rods, thence to the left 5 rods, thence to the place of beginning.

Find (a) the distance around the field, (b) the area in acres, and (c) the cost of fencing the field, the posts to cost 10 cents each, set $\frac{1}{2}$ rod apart, the fence to be of boards $16\frac{1}{2}$ feet long and 4 boards high, the lumber costing \$8 per M.

3. Find the amount of inch lumber required to make a box 4 feet wide, 6 feet long, 3 feet high, outside measurement. Find the outside surface. Find the inside surface.

4. Find the amount of 2-inch lumber required for a box having the same outside dimensions. Find the inside surface.

5. Find the amount of inch lumber required for a box 4 feet wide, 6 feet long, 3 feet high, inside measurement. Find the outside surface.

6. Find the amount of 2-inch lumber required for a box having the same interior dimensions. Find the outside surface.

7. If a business yield \$700 net profit in 1 year 8 months, in what time will the same business yield \$1,050 profit?

8. If a pole $12\frac{1}{2}$ feet long casts a shadow $9\frac{3}{4}$ feet long, how long must a pole be to cast a shadow 21 feet long at the same hour of the day?

9. If 5 men can do a piece of work in 4 hours 30 minutes 20 seconds, how long will it take 8 men to do the work?

10. If 25 men can do a certain piece of work in $27\frac{1}{2}$ days, how many days will it take 5 men to do it? By proportion.

11. A farmer sold 180 bushels of corn at $\$.37\frac{1}{2}$ a bushel and 172 bushels of rye at $\$.87\frac{1}{2}$ a bushel, and with the money bought a span of horses worth \$225. How much money had he left?

12. How much cloth $\frac{3}{4}$ of a yard wide will line $6\frac{3}{4}$ yards $1\frac{1}{2}$ yards wide?

PERCENTAGE.

PERCENTAGE, as an operation, is a process of computation, in which the basis of comparison is one hundred.

THE RATE per cent is the number of hundredths taken.

THE BASE is the number of which the hundredths are taken.

THE PERCENTAGE of a number is the result found by taking the part of it denoted by the rate per cent.

THE RATE per cent being a number of hundredths, is a *fraction*, and may be expressed in the form of a decimal or a common fraction.

The three elements in a problem in percentage are the *base*, the *rate*, and the *percentage*. Since the percentage is the product of the base and the rate, it is evident that if two of these elements are given, the third may be found.

1. Find 25% of 500.

SOLUTION—BY ANALYSIS.

100% = 500,
1% = $\frac{1}{100}$ of 500, or 5,
25% = 25×5, or 125.

SOLUTION—BY COMMON FRACTIONS.

25% = $\frac{1}{4}$ of the whole,
$\frac{1}{4}$ of 500 = 125.

SOLUTION—BY DECIMAL FRACTIONS.

25% = .25 of the whole,
.25×500 = 125.

SOLUTION—BY PROPORTION.

100% : 25% :: 500 : x,
Solving, x = 125.

Find—

2. 10% of 1500. **5.** 20% of 1500. **8.** $3\frac{1}{2}$% of \$2600.
3. 6% of 1200. **6.** 25% of 1600. **9.** 4 % of 365 days.
4. 8% of 125. **7.** 40% of 3300. **10.** $\frac{1}{2}$% of \$3600.

1. Find what per cent 10 is of 25.

SOLUTION—BY ANALYSIS.

$25 = 100\%$,
$1 = \frac{1}{25}$ of 100%, or 4%,
$10 = 10 \times 4\%$, or 40%.

SOLUTION—BY COMMON FRACTIONS.

$1 = \frac{1}{25}$ of 25,
$10 = \frac{10}{25}$ of 25, or $\frac{2}{5}$ of 25,
$\frac{2}{5}$ of $100\% = 40\%$.

SOLUTION—BY DECIMAL FRACTIONS.

$25 = \frac{100}{100}$ of itself,
$1 = \frac{1}{25}$ of $\frac{100}{100}$, or .04 of 25,
$10 = 10 \times .04 = .40$, or 40% of 25.

SOLUTION—BY PROPORTION.

$25 : 10 :: 100\% : x\%$. Solving, $x = 40\%$.

2. Find a number of which 30 is 5%.

SOLUTION—BY ANALYSIS.

$5\% = 30$,
$1\% = \frac{1}{5}$ of 30,
$100\% = 100 \times \frac{1}{5}$ of 30, or 600, the number.

SOLUTION—BY COMMON FRACTIONS.

$5\% = \frac{5}{100}$ of a number,
$\frac{5}{100} = 30$,
$\frac{20}{20} = 20 \times 30 = 600$, the number.

SOLUTION—BY DECIMAL FRACTIONS.

$5\% = .05$ of a number,
$.05$ of a number $= 30$,
The number $= 30 \div .05 = 600$.

SOLUTION—BY PROPORTION.

$5\% : 100\% :: 30 : x$. Solving, $x = 600$.

3. Find the number of bushels of which 16 quarts are 5%.

4. Find what per cent of 6 gallons are 6 pints.

1. A horse cost \$240. If it was sold at a gain of 25%, how many dollars were gained by selling it?

SOLUTION I.

1 per cent of the cost = \$2.40 gain,
25 per cent of the cost = 25 × \$2.40 or \$60 gain.

SOLUTION II.

25 per cent = .25 of the cost,
.25 times \$240 = \$60 gain.

SOLUTION III.

25 per cent = $\frac{1}{4}$ of the cost,
$\frac{1}{4}$ of \$240 = \$60 gain.

SOLUTION IV.

100 per cent : 25 per cent :: = \$240 : gain,
Solving the proportion, gain = \$60.

What is—

2. $12\frac{1}{2}$% of 128 cows?
3. 50 % of 320 bales?
4. $37\frac{1}{2}$% of 1275 yards?
5. 95 % of \$4573?
6. $87\frac{1}{2}$% of 2500 bbls?
7. 112% of \$4573?
8. 25 % of \$5000?
9. $318\frac{1}{3}$% of \$1267.87$\frac{1}{2}$?
10. 5000% of \$3000?
11. $133\frac{1}{3}$% of \$765?
12. $31\frac{1}{4}$% of 960 bushels?
13. $62\frac{1}{2}$% of a sec. of land in acres?

14. A tenant received $37\frac{1}{2}$% of a crop of wheat from a field of 136 acres, which averaged 35 bushels to the acre. Find the value of his share at \$.37$\frac{1}{2}$ a bushel.

15. A lot in a city was bought for \$250, and increased 125%. What was it then worth?

16. A daily paper had a circulation of 65,000 copies, and a weekly circulation of 30% more. Find the number of copies of the weekly edition.

17. A man borrowed \$2,500 for one year, and paid 6% interest for its use. What amount was due at the end of the year?

The *amount* is the sum of the base and percentage.

The *difference* is the remainder after substracting the percentage from the base.

1. From 18 bushels of wheat sown, a crop of 288 bushels was harvested. The seed was what per cent of the yield?

SOLUTION.

288 bu. = 100 per cent,
1 bu. = $\frac{100}{288}$ of 100 per cent,
18 bu. = $\frac{1800}{288}$ of 100 per cent, or $6\frac{1}{4}$ per cent.

2. Of the above wheat crop, what per cent of the seed was the yield?

SOLUTION.

18 bu. = 100 per cent,
1 bu. = $\frac{100}{18}$ of 100 per cent,
288 bu. = $\frac{28800}{18}$ of 100 per cent = 1600 per cent.

3. What per cent of 680 is 119?

4. Property costing $6,500 rents for $617.50 a year. What per cent of its cost is the rent?

5. What number increased by 20% of itself equals 360?

SOLUTION I.

100 per cent = the number,
100 per cent + 20 per cent = 120 per cent,
120 per cent = 360,
1 per cent = 3,
100 per cent = 300, the number required.

SOLUTION II.

20 per cent = $\frac{1}{5}$,
$\frac{5}{5} + \frac{1}{5} = \frac{6}{5}$, = 360,
$\frac{1}{5} = \frac{1}{6}$ of 360, or 60,
$\frac{5}{5}$ = 300, the number required.

6. What number plus 16% of itself equals 435?

7. How much money less 78% of itself leaves $44?

8. What number less 5% of the number equals 57?

9. What number plus 7% of the number equals 642?

10. What number diminished by 20% of itself equals 320?

11. What number diminished by 5% of itself equals 912?

12. What sum diminished by 25% of itself equals $20.50?

1. What number increased by $8\frac{1}{3}\%$ of itself equals 6,500?

2. What number increased by $6\frac{1}{4}\%$ of itself equals 8,500?

3. What number diminished by $12\frac{1}{2}\%$ of itself equals \$1,400?

4. What number diminished by 5 % of itself equals 7,600?

5. In a township of land surveyed for settlement in Oregon, $62\frac{1}{2}\%$ of the township was timber land. How many acres not timbered?

6. A farmer raised a crop of 812 bushels of grain of which $12\frac{1}{2}\%$ was barley and the remainder was wheat. How many bushels of wheat did he raise?

7. Of a regiment of 1,000 men sent to the Philippine Islands, $12\frac{1}{2}\%$ was killed, 10% remained at Manila, and the remainder returned to America. How many returned?

8. Of 8,500 tons of coal shipped from Puget Sound to California, 25% was from Newcastle, 56% was from Carbonado, and the rest from Black Diamond. How many tons from each place?

9. A speculator bought 200 acres of land, which was $16\frac{2}{3}\%$ of what he already owned. He then sold 900 acres. The remainder was what per cent of what he had at first?

10. From 760 pounds of sugar 190 pounds were sold at one time and 380 pounds were sold at another time. What per cent of the first amount remained?

11. Of 1,200 acres of land $\frac{1}{8}$ of it was sold to one man and $\frac{1}{4}$ of the remainder to another. What per cent remained unsold?

12. A school had an average attendance of 230 pupils, which was 92% of the number enrolled. How many pupils were enrolled?

13. In a baseball league a club won 55 games and lost 11 games. What per cent of the games played were won? Were lost?

Give an original problem to show—

14. How to find the percentage, when the base and rate are given.

15. How to find the rate, when the base and percentage are given.

APPLICATIONS OF PERCENTAGE.

The principal applications of Percentage, in which *time* is not an element, are *Profit* and *Loss*, *Commission* and *Brokerage*, *Trade Discount*, *Insurance*, *Taxes*, *Stocks*, and *Custom House Business.*

Those in which *time* is an element are *Interest*, *Discount*, *Exchange*, *Equation of Accounts*, and *Investments.*

Profit and loss are commercial terms used to express the gain or loss in business transactions, and are usually estimated at a *rate per cent* on the *cost*, or the money *invested.*

1. A horse that cost \$200 was sold at a gain of 5%. Find the gain and the selling price.

SOLUTION I.

100 per cent = \$200, the cost,
1 per cent = \$2,
5 per cent = \$10, the gain,
\$200+\$10 = \$210, the selling price.

SOLUTION II.

100 per cent : 5 per cent :: \$200 : gain,
100 per cent : 105 per cent :: 200 : selling price.

SOLUTION III.

5 per cent = $\frac{1}{20}$, $\frac{1}{20}$ of \$200 = \$10, the gain,
\$200+\$10 = \$210, selling price.

2. A boy gave \$20 for a bicycle, and sold it for \$25. What per cent did he gain?

SOLUTION I.

100 per cent = \$20, 1 per cent = \$.20, \$5, the gain, is such per cent gain as \$.20 is contained times in \$5, or 25 per cent.

SOLUTION II.

100 per cent : gain per cent :: \$20 : \$5 (gain).

SOLUTION III.

\$25—\$20 = \$5, the gain,
\$5 = $\frac{1}{4}$ of \$20,
$\frac{1}{4}$ of 100 per cent = 25 per cent.

COMMISSION is an allowance or compensation to an agent, commission merchant, or factor. Commission merchants are usually placed in possession of the goods bought or sold.

The business of a *Broker* is similar to that of a commission merchant. Brokers do not have possession of the merchandise bought or sold.

The term *broker* is more generally applied to those whose business it is to buy and sell stocks, bonds, notes, etc.

The person for whom the business is transacted is called the *Principal.*

The person who sends the goods to be sold is called the *Consignor* or *Shipper.*

THE CONSIGNMENT is the goods sent.

The person to whom the goods are sent is called the *Consignee.*

THE NET PROCEEDS of a consignment is the balance due the *consignor* after charges and expenses have been deducted.

In *selling*, the commission is reckoned on *the money received;* in *buying*, the commission is reckoned on *the money paid.*

Find an agent's commission—

1. At 3 % for selling $12,500 worth of goods.
2. At 2½% for buying $25,000 worth of goods.
3. At 6 % for collecting $4,500 worth of outstanding bills.
4. At 2 % for selling $80,000 worth of grain.
5. At 3 % for selling $25,000 worth of farm machinery.
6. For selling $40,000 worth of goods, at 3%.
7. For selling $75,000 worth of produce, at 1½%.
8. For selling $ 1,550 worth of fruit, at 2½%.
9. For collecting $23,000 taxes, at ½%.
10. For investing $3,500, at 2½%, and $2,500, at 2%.
11. For selling $64,000 of bank stock, at ⅜%.
12. For buying a $1,000 share in a publishing business at 1¾%.
13. For selling 12 horses at $250 each, at 2½%.
14. For selling 5 acres of land at $420 an acre, at 3½%.
15. For buying 1,000 bu. of wheat, at 75¢ a bu., at ½%.

When a reduction is made from the marked or assumed price of an article, from the amount of a debt, or from the face of a note, it is called a *Discount.* Some kinds of merchandise—books, furniture, musical instruments, etc.,—manufacturers and wholesale dealers invoice to the trade, or retail dealers, at the fixed or *list prices.*

The *List Prices* are usually the retailer's selling prices.

The manufacturer or wholesale dealer allows the retailer a *Trade Discount,* which is deducted at a certain rate per cent from the face of the invoice.

Very often *two or more discounts are deducted in succession.* Thus 10% and 5% off—or as it is generally expressed in business, 10 and 5 off—means a discount of 10%, and then 5% from what is left; 20, 10, and 5 off means three successive discounts. A retailer's profit is smaller when he is allowed 10% and 5% off than if he were allowed 15% off. The result is not affected by the order in which the discounts are taken.

1. Goods are marked at $100. What will be the selling price if they are discounted 20% and 10% off the marked price?

SOLUTION I.

20 per cent of $100 = $20,
$100 — $20 = $80,
10 per cent of $80 = $8,
$80 — $8 = $72, the cost.

SOLUTION II.

A discount of 20 per cent leaves 80 per cent of $100 = $80,

A discount of 10 per cent leaves 90 per cent of $ 80 = $72, the cost.

SOLUTION III.

100 per cent — 20 per cent = 80 per cent,
100 per cent — 10 per cent = 90 per cent,
80 per cent of 90 per cent of $100 = $72, the cost.

2. What is the cost of a bill of goods listed at $1,000, sold at a discount of 20 and 10 off.

STOCK, OR STOCKS, is the name given to certificates which show that the holder owns one or more shares in a business corporation.

A CORPORATION is an organization of several persons who are authorized by law to transact business as a single individual.

Suppose a city desires to have a water system, and that $50,000 will be needed to construct the works. No one person having so much money, it becomes necessary to have several persons take *Shares* in the enterprise by putting in $100 or more. The amount necessary to be raised, known as the *Capital Stock*, can thus be secured. The *stock* of a corporation usually consists of *shares* of $100 each.

After the *capital stock* is all subscribed, the subscribers obtain legal authority to act as a company, and elect a *Board of Directors*, each subscriber usually having one vote for each share of stock subscribed. The *board of directors* then calls for the payment of the subscriptions as the money is needed, until they are all paid.

A CERTIFICATE OF STOCK is then given to each *Stockholder*, showing the number of shares he has taken, and the value of each share. This value named in the *certificate* is called the *face*, or *par value*.

If the business is successful, and pays in *Dividends*, or profits, more than the money could earn in other investments, the stock will be at a *Premium;* that is, its *market value* will be more than its *par value*. But if the dividends are smaller than the income from other investments, the market value will be less than $100 a share, and the stock will be below par, or at a *Discount*.

DIVIDENDS are declared yearly, half-yearly, or quarterly, at a certain rate per cent of the par value of the stock. The amount divided is the *net earnings*, or the amount remaining after deducting the expenses from the total receipts.

AN ASSESSMENT is a sum required of stockholders, to meet the losses, or to pay the business expenses of the company. It is also estimated at a certain per cent of the par value of the stock.

A STOCK BROKER is one who buys and sells stocks for others. For buying or selling stocks in the New York Stock Exchange, the regular charge is $\frac{1}{8}$ of 1% on their *par value*.

A STOCK JOBBER buys and sells stocks on his own account.

1. What is the market value of 110 shares of stock when sold at 5% above par?

2. What is the market value of 120 shares of stock when sold at 5% below par?

3. What is the cost of 12 shares of stock at 5% above par, if I pay a stock broker $\frac{1}{8}$% for buying it for me?

4. What is the cost of 12 shares of stock at 5% below par, if I pay a stock broker $\frac{1}{8}$% for buying it for me?

5. A company with a capital of $120,000, gained in one year, $10,000 above all expenses. What per cent dividend was made to each stockholder?

6. I wish to insure my home, which is worth $4,000. I find that I must pay to an insurance company 1% annually on the amount I wish insured. What will be the cost to insure the house for full value? For $\frac{3}{4}$ of its value?

INSURANCE is indemnity secured against possible loss or damage. It is of two general kinds, *insurance on property* and *insurance on life.*

PROPERTY INSURANCE includes *fire insurance*, or indemnity for loss of property by fire; *marine insurance*, for loss of vessel or cargo, whether at sea or on inland waters, and *live stock insurance*, for loss of horses, cattle, etc.

THE POLICY is the contract, or agreement, between the insurer and the insured.

THE PREMIUM is the sum paid for insurance, and is a certain *percentage* of the amount insured.

7. A stock of goods worth $15,000 is insured for $\frac{4}{5}$ its value at 1%. What is the annual cost, or premium?

8. I paid $50 for insuring $\frac{1}{2}$ my property at $\frac{1}{2}$%. What is the property worth?

9. What is the premium on $2,400 at $\frac{3}{8}$%? At $\frac{3}{4}$%? At $1\frac{5}{8}$%?

1. An amount of sugar, bought for \$54.40, was sold to gain $12\frac{1}{2}\%$. What was the profit?

2. A horse cost \$175, and was sold to gain 15%. What was the selling price?

3. Find the gain per cent on flour costing \$7 a barrel, and selling for \$8 a barrel.

4. Find the loss per cent in selling a farm for \$2,485.84, which cost \$2,576.

5. Find the cost of a city lot sold for \$205, at a loss of 18%.

6. Find the profit on 60 yards of muslin, bought for 20 cents a yard, and sold to gain $33\frac{1}{3}\%$.

7. Albert bought a boat for \$850, sold it to his brother at a gain of 25%, and his brother sold it back at a loss of 20%. What did Albert pay for it the second time?

8. A tank containing 600 gallons of water receives by a pipe 10% of its capacity in an hour, and by another pipe it loses 18%. How much water remains in the tank after both pipes are open for an hour?

9. A dealer bought goods for 20% less than the marked price, and sold them at a profit of 20%. The selling price was what per cent more or less than the marked price?

10. A dealer bought goods for 20% less than the marked price, and sold them at the marked price. Find the gain per cent.

11. A residence property was sold for \$33,744, at a gain of 14%. For what would it have sold to lose 14%?

12. A book dealer lost 16% by selling his stock for \$1,920 less than cost. Find the selling price that he might gain 16%.

13. Two merchants each lost \$220 worth of goods in a railroad wreck, which was 20% of the stock of one, and $5\frac{1}{2}\%$ of the stock of the other. Find the value of each merchant's stock.

14. A real estate agent sold two pieces of property for \$12,180 each. On one he gained 16%, and on the other he lost 16%. Find the gain or loss on the two sales.

15. Find the cost of a house sold for \$750 at a loss of $33\frac{1}{3}\%$.

Give an original problem requiring—

1. The *cost*, when the *rate* and the *gain* or *loss* are given.
2. The *rate*, when the *cost* and the *gain* or *loss* are given.
3. The *gain* or *loss*, when the *rate* and the *cost* are given.
4. The cost, when the *rate* and the *selling price* are given.

5. Residence property was sold for \$7,500, at a gain of \$1,500. If the gain had been twice as much, what per cent profit would have been made?

6. A merchant lost 15 gallons of oil by leakage from a cask of 75 gallons. What per cent increase on the remaining gallons is necessary to prevent loss? What will be the price of a gallon if the oil cost \$.50 a gallon?

7. If an agent sells an acre of land for \$175 and gains 5%, what would have been his gain per cent by selling it for \$200?

8. A merchant gained 5% by selling flour for \$8.40 a barrel. What would have been his gain per cent by selling for \$9.50 a barrel?

9. A farm was sold for \$5,000, at a gain of 25%. For what would it be sold at a loss of 25%?

10. A piece of land was bought for \$24 an acre. For how much a square rod must it be sold to gain 20%?

11. A merchant bought some goods for \$1,500, which was \$500 less than the real value. He sold them at a gain of 25% on their value. What was his gain per cent on his investment?

12. A farmer sold $\frac{3}{4}$ of his farm for what the whole farm cost him. What was the gain per cent on the part sold?

13. An acre of land 8 rods by 20 rods cost \$240. A piece 8 rods by 12 rods was sold for the cost of the whole acre. At that rate, what per cent gain would be made on the whole acre?

14. A grocer bought 2 bushels of nuts at \$8 a bushel, and sold them for 15 cents a pint. What was his gain per cent?

15. A grocer bought 200 crates of strawberries, each containing 21 boxes, for 10 cents a box. He lost 800 boxes in handling, and sold the rest so as to make $12\frac{1}{2}$%. What was the price per box?

Find the wholesale price at—

1. A discount of 40, 10, and 5, from list price of $1,384.90.

2. A discount of 25, 20, and 1, from list price of $1,608.40.

3. A discount of 25, 20, and 10, from list price of $ 402.25.

4. A discount of 20, 10, and $3\frac{1}{3}$, from list price of $ 808.00.

5. A discount of $16\frac{2}{3}$, $12\frac{1}{2}$, and 10, from a list price of $2,790.

6. An agent bought a residence property for $9,780, and was paid $2\frac{1}{4}\%$ commission. What did he receive?

7. A collector sent to his employer $6,550 for his month's collections. His commission was $32.75. What was his rate of commission?

8. In one month an agent received $155.50 for his commission at 20%. What amount of business did he transact?

9. An agent sold a half-section of land at $75 an acre, at $\frac{3}{8}\%$ commission. How much did he receive? What amount did he send to the owner of the land?

10. A collector was paid $105 for collecting $1,750. What was his rate of commission for collecting?

11. An agent bought flour for $4.50 a barrel, and sold it for $6 a barrel. Find his commission on 5,000 barrels, if he receives $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling.

12. Find the commission of an agent who sold a house for $4,550, at 2% commission. What did the principal receive?

13. An auctioneer sold $3,950 worth of goods at a sale, and received $2\frac{1}{2}\%$. Find his commission.

14. A commission merchant received from a farmer 400 boxes of apples worth $1.50 a box, 500 bushels of potatoes worth $.37$\frac{1}{2}$ a bushel, and 50 tons of hay worth 11\frac{1}{2}$ a ton. What was the commission for selling at $1\frac{1}{2}\%$, and what was the farmer's net proceeds?

15. A store is insured for $\frac{4}{5}$ its value, at 1%. What is the value of the store, if the premium is $480?

16. Write an original problem in the commission business.

1. Find the cost of 100 shares at 90, brokerage $\frac{1}{8}\%$.

SOLUTION.

Market price per share = 90 per cent of par value,
Brokerage per share = $\frac{1}{8}$ per cent of par value,
Total cost per share = $90\frac{1}{8}$ per cent of face value, or $\$90.12\frac{1}{2}$,
100 shares will cost $100 \times \$90.12\frac{1}{2}$, or \$9,012.50.

2. Find the annual income from 50 shares of bank stock, if a 2% dividend is paid every 3 months.

3. Find cost of 50 shares stock, par value, brokerage $\frac{1}{2}\%$.

4. Find cost of 20 shares railroad stock, at 105, brokerage $\frac{1}{8}\%$.

5. Find cost of \$30,000 U. S. bonds, at 112, brokerage $\frac{1}{8}\%$.

6. How many shares can be bought with \$2,505, at 25% premium, brokerage $\frac{1}{4}\%$?

SOLUTION.

Market value = \$125.00 per share,
Brokerage = .25,
Total cost = \$125.25 per share,
\$2,505 will buy as many shares as \$125.25 is contained times in \$2,505.25, or 20 shares.

Find the number of shares which can be bought with—

7. \$26,000 invested in railroad stock, at $25\frac{3}{4}$, brokerage $\frac{1}{4}\%$.

8. \$ 2,350 invested in U. S. bonds, at $117\frac{3}{8}$, brokerage $\frac{1}{8}\%$.

9. \$32,500 invested in U. S. 5's, at 30% premium, no brokerage. What is the annual income from the investment?

10. What income is derived from investing \$2,650 in 5% bonds, at $105\frac{3}{4}$, brokerage $\frac{1}{4}\%$?

11. Find the income from investing \$6,900 in $4\frac{1}{2}\%$ stock at 86, brokerage $\frac{1}{4}\%$.

12. A stock jobber has \$12,075 to invest. Which will pay better, to buy State bonds, bearing 6% for 105; or to buy Chicago 7's at 115? How much better?

13. Find the number of shares which can be bought for \$2,775, at $110\frac{1}{2}$, brokerage $\frac{1}{8}\%$, and the annual income from a 3% quarterly dividend.

1. If a broker buys stock at 60 and sells it at 90, he makes \$240. How many shares did he buy?

SOLUTION.

Cost = 60 per cent of par value,
Selling price = 90 per cent of par value,
90 per cent − 60 per cent = 30 per cent gain,
30 per cent = \$240,
1 per cent = \$8,
100 per cent = \$800, amount of stock purchased,
or 8 shares.

2. Street railway stock was bought at 90, and sold at par. The gain was \$1,000. How many shares bought?

3. How many shares of stock must be bought at 110, to lose \$1,200 by selling at 98?

4. The loss on mining stock bought at par and sold at 96 was \$4,000. How many shares were bought?

5. A broker bought stock at 8% discount, and sold at 10% premium, gaining \$360. How many shares did he buy?

6. What will cost U. S. 6's, at 112, to make an annual income of \$1,200?

SOLUTION.

The income from 1 share = \$6. To produce an income of \$1,200, it will take as many shares as \$6 is contained times in \$1,200, or 200 shares. Market price = \$112 per share. 200 shares will cost 200×\$112, or \$22,400.

7. Find the amount invested in 10% stock, at 115, to produce an annual income of \$2,000.

8. Find the amount invested in 5% stock, at 103, to produce an annual income of \$800.

9. What amount invested in U. S. 4½'s, at 105, will secure \$900 annually?

10. How much must be invested in railroad bonds bearing 7%, at 106¾, brokerage ¼%, to make \$350 per annum?

1. What per cent is made by investing in 8% stock at 160?

SOLUTION.

Cost of 1 share = \$160,
Income from 1 share = \$8,
\$8 is $\frac{1}{20}$ or 5 per cent of \$160, the amount invested.

2. What per cent is made by buying 6% stock at 120?
3. What per cent is made by buying 5% stock at 125?
4. What per cent is made by buying 4% stock at 115?
5. Which is the better investment, State 6's at 120, or R. R. 7's at 104?
6. Which is the better investment, 6% bonds at 80, or 5% bonds at $66\frac{2}{3}$?
7. If a bond costing 108 pays \$4.50 annual interest, what rate is received on the investment?
8. What must be paid for 6% stock, to make 8% on the amount invested?

SOLUTION.

The income = \$6 from each share,
8 per cent of the cost of each share = \$ 6,
1 per cent of the cost of each share = \$ .75,
100 per cent of the cost of each share = \$75.

9. What must be paid for 7% stock to make 5% on the investment?
10. What must be paid for Missouri 6's to make 5% on the cost?
11. What must be paid for 5% stock to make 6% on the investment?
12. What must be paid for 8% stock to make 10% on the investment?
13. At what price must 4% bonds be bought to make 5% on the cost?

NOTE.—**In buying, the brokerage is added to the cost; in selling, it is subtracted from the selling price.**

Find the premium for insuring—

1. A store worth 7,500, at $2\frac{1}{4}\%$.
2. A factory worth 12,000, at $2\frac{3}{8}\%$.
3. A house worth 3,600, at $2\frac{1}{2}\%$.
4. A cargo of goods worth 12,000, at $\frac{7}{8}\%$.
5. A mill worth 24,000, for $\frac{3}{5}$ its value, at $1\frac{3}{4}\%$.
6. A cargo of 9,680 bu. of wheat, worth 80¢ a bu., at $1\frac{7}{8}\%$.
7. A factory worth 72,000, for $\frac{2}{3}$ the value, at $3\frac{1}{3}\%$.
8. A ship and cargo worth $46,370, at $2\frac{1}{4}\%$.
9. A residence and furniture worth $15,000, at $2\frac{1}{3}\%$.
10. A business block worth $90,000, for $\frac{3}{4}$ its value, at $\frac{1}{4}\%$.
11. The policy is $20,000, the premium is $350. Find the rate of insurance.
12. A Pacific steamship and cargo, worth $450,000, was insured for $\frac{3}{5}$ its value, paying a premium of $6,750. What was the rate?
13. A premium of $75 was paid to insure a herd of cattle, at the rate of 30 cents per $100. What was the value of the herd?
14. A premium of $60.75 was charged on a policy of $2,700. What was the rate?
15. A premium of $1,443.75 was charged to insure a cargo of 5,000 barrels of flour, worth $5.25 a barrel. What was the rate?
16. A gentleman's life insurance policy for $15,000 cost him annually $525. What was the rate of premium?
17. The premium for insuring a house at $\frac{1}{3}\%$ was $20. Find the face of the policy.
18. It cost $150 to insure a hotel for $\frac{2}{3}$ its value. The rate was 75¢ a $100. What was the value of the hotel?
19. A merchant imported 150 hogsheads of molasses, at 35 cents a gallon, and gets it insured for $3\frac{1}{3}\%$ on the selling price of 50 cents a gallon. If the whole should be destroyed, and he gets the amount of insurance, how much would he gain?
20. If I get my house and furniture, valued at $3,640, insured at $4\frac{1}{2}\%$, what would be my actual loss if they were destroyed?
21. Define insurance, policy, and premium.

TAXES are sums of money assessed on property, business or incomes for public purposes.

REAL ESTATE is fixed property, as lands and houses.

PERSONAL PROPERTY is movable property, as money, goods, furniture, ships, cattle, stocks, etc.

AN ASSESSOR is an officer appointed to estimate the value of property.

A PROPERTY TAX is a tax upon property, and is assessed at a given rate. per cent of the valuation, or so many cents on the $100, or mills on the dollar.

1. The property of a certain state was assessed at a valuation of $227,000,000. A tax of 2.4 mills on a dollar was levied to pay the current expenses of the state, and 2.6 mills was levied for the annual state school fund. What was the total amount of state tax paid?

2. The property in one of the counties of this state was assessed at a valuation of $5,500,000. What amount of tax did the county pay into the state school fund?

3. A certain town of this state having property assessed at $800,000, in addition to the state tax above mentioned, raised 12 mills on the dollar for local and county purposes. What was the total amount of tax paid?

4. The taxable property of a town was $1,100,000. A tax of $13,200 was levied to build a water system. What was the rate of taxation? How much tax did Mr. Allen pay on property valued at $35,000?

5. Find Mr. Chase's tax on property valued at $7,500, if he pays 5 mills town tax, 6 mills county tax, 3 mills state tax, and $1\frac{1}{2}$ mills special school tax.

6. What is the rate of taxation, if $7,200 is raised on property valued at $1,800,000?

7. A town levied a $2\frac{1}{4}$ mill tax to build a bridge which cost $1,833.30. What was the assessed valuation of the property in the town?

8. Make an original problem in taxes.

1. Find the number of shares of New York Central railroad stock, at $106\frac{7}{8}$, brokerage $\frac{1}{8}\%$, which can be bought for \$21,400.

2. Find what amount must be invested in State 6's, at 90, to make an annual income of \$24,000.

3. Find the difference between a direct discount of 35%, and a successive discount of 20, 10, and 5.

4. A retailer buys \$180 worth of goods, and from one wholesale house he is offered a discount of 20, 10 and 5 off, and by another he is offered a discount of 25 and 10 off. Which is the better bargain, and how much better?

5. An agent sold four pianos for \$200, \$300, \$400, and \$500, for which he had paid 20 and 10 off from the selling price. For what should he have sold each one to make 50%?

6. A book dealer bought a set of Shakespeare's works for 20 and 10 off the catalogue price, and sold them for 10% above catalogue price. What was his per cent profit?

7. A music dealer bought a set of musical instruments for 40 and 5 off the list price. What would he have gained by paying 45 off, when the instruments were listed at \$640?

8. Give an original problem, showing what, if any, difference there is between 20 and 5 off and 5 and 20 off the marked price of goods.

9. How many \$500 U. S. 5's must I buy to have an annual income of \$1,250? How much will they cost when quoted at $104\frac{1}{2}$?

10. How much must be invested in N. Y. 7's, at 120, to secure a semi-annual income of \$1,400?

11. How many shares bought at $97\frac{3}{8}$, and sold at $107\frac{5}{8}$, brokerage $\frac{1}{4}\%$ for each transaction, will make a profit of \$850?

12. A broker buys a share of stock bearing 6% for \$80, and a share at par bearing $7\frac{1}{2}\%$. Which produces the better annual income? By selling both shares at par in one year, what is his gain or loss?

13. At what must 5% stock be bought to make as good an investment as 8% stock at 160?

14. Make an original problem in stocks.

1. \$20 is $12\frac{1}{2}\%$ of what?
2. 18 is $\frac{1}{2}\%$ of what?
3. 18 s. is $\frac{1}{2}\%$ of what?
4. 34 qts. is $8\frac{1}{2}\%$ of what?
5. 15 cts. is .01% of what?
6. $2\frac{1}{4}$ lbs. is $\frac{1}{4}\%$ of what?
7. 1 s. is $3\frac{1}{3}\%$ of what?
8. \$40 is 150% of what?
9. 112 is 14% of what?
10. \$52 is .3% of what?
11. 1800 is 40% of what?
12. 14.4 bu. is $2\frac{2}{5}\%$ of what?
13. 913.5 is 105% of what?
14. 1% of 240 is 80% of what?
15. $\frac{1}{2}$ of 90 is 300% of what?
16. $87\frac{1}{2}$ is $12\frac{1}{2}\%$ of what?
17. $\frac{7}{8}$ is 45% of what?
18. $\frac{3}{4}$ is $\frac{5}{6}\%$ of what?

19. A ship valued \$20,450, and her cargo at \$25,600, was bound on a voyage. The vessel was insured for \$12,000, at $2\frac{3}{4}\%$, and the cargo for \$18,500, at $3\frac{1}{4}\%$. The vessel was lost at sea. What was the entire loss of the owner?

20. A merchant paid \$1,320 insurance on his vessel and cargo, which was $5\frac{1}{2}\%$ on the amount insured. How much did he insure?

21. A man pays \$51 a year for insurance on his storehouse, at $1\frac{1}{2}\%$, and \$126.45 on the contents, at $2\frac{1}{4}\%$. What amount of property does he get insured?

22. A merchant owned a steamboat worth \$64,000. He had $\frac{5}{8}$ of it insured by one company at $\frac{3}{4}\%$, and $\frac{3}{8}$ of it at $\frac{7}{8}\%$. Find the average rate of premium on the whole insurance.

23. A person receives \$5,850, which is 5% of what is due him. What is the amount due him?

24. A farmer keeps 25% of his sheep in one field, and 15% in another, and the remaining number, 48, in a third. How many sheep had he?

25. A collector, who gets 3% for his services, makes \$33.33 by collecting a certain bill. How much must he pay over to his employer?

26. Mr. Wilson lost 18% of his sheep by disease, and had a flock of 615 left. How many had he at first?

27. Mr. Jones invests 46% of his capital in land, and has \$513 left. What was his capital?

28. Make an original problem in percentage.

1. Find the rate of commission for selling \$2,780 worth of goods for which the commission was \$69.50.

2. Find the annual sales of a merchant that he may realize \$4,200 by a profit of 35%.

3. The surplus fund of an insurance company, amounting to \$32,500, will pay 12½% on its capital. What is the capital?

4. The profits of a bank are 12% of the capital; the expenses are 10% of the profits. What per cent of the capital are the expenses?

5. A profit of 4% is realized by selling some goods for \$288.80. What per cent would have been gained or lost by selling them for \$215.60?

6. By selling some grain for \$1,335, 11% was lost. What per cent would have been gained or lost by selling it for \$3,000?

7. A profit of 12⅝% was made by selling goods for \$1,537.90. What per cent would have been gained or lost by selling them for \$1,751.65?

8. A collector received 4½% for collecting a town tax of \$2,564.25. What was his percentage?

9. An attorney received 6⅝% for collecting a debt of \$7,320.25. What did his client receive?

10. A dairyman sent to a commission merchant 3,472 pounds of cheese. The commission for selling it was 3½%. What did the commission merchant remit to the dairyman, after selling the cheese for 12½ cents a pound?

11. A real estate agent received \$175 for selling a house and lot for \$7,000. What was his rate of commission?

12. A collector received \$13.50 for collecting \$175. What was the rate of his commission?

13. When wheat sells for \$.75 a bushel at Cincinnati it sells for \$.87½ at Liverpool. (*a*) What per cent is the Liverpool price above the Cincinnati price? (*b*) What per cent is the Cincinnati price lower than the Liverpool price?

14. What per cent less than 80 is 60? What per cent more than 60 is 80?

As *wages* is the sum of money paid for labor, and *rent* is the sum paid for the use of property, so *interest* is the sum paid for the use of money.

THE PRINCIPAL is the sum for the use of which interest is paid.

THE RATE OF INTEREST is the per cent of the principal charged for its use for *one year*.

LEGAL INTEREST is the interest reckoned at the rate per cent fixed by law.

SIMPLE INTEREST is interest on the principal only, for the full time.

1. What is the interest of \$50 at 6% for 3 years?

SOLUTION I.

Interest at 6 per cent for 1 year is .06×\$50, or \$3,
Interest at 6 per cent for 3 years is 3× \$3, or \$9.

SOLUTION II.

Interest at 1 per cent for 1 year = \$.50,
Interest at 1 per cent for 3 years = 3×\$ .50, or \$1.50,
Interest at 6 per cent for 3 years = 6×\$1.50, or \$9.

SOLUTION III.

Interest on \$ 1 at 6 per cent for 1 year is \$.06,
Interest on \$ 1 at 6 per cent for 3 years is 3×\$.06, or \$18,
Interest on \$50 at 6 per cent for 3 years is 50×\$.18, or \$ 9.

In each solution it is seen that the interest is the product of three factors: *Principal*, *rate* and *time*.

2. What is the interest on \$50 at 6% for 3 years 4 months?

SOLUTION.

Interest of \$50 at 6 per cent for 1 year = \$3,
Interest of \$50 at 6 per cent for 3 years = 3×\$3 = \$9,
Interest of \$50 at 6 per cent for $\frac{1}{3}$ year = $\frac{1}{3}$×\$3 = \$1,
Interest of \$50 at 6 per cent for 3 years 4 months = \$10.

What is the interest—

3. Of \$600 for 2 years 3 months at 8%?

4. Of \$450 for 4 years 6 months at 6%?

5. Of \$250 for 3 years 9 months at 8%?

6. Of \$650 for 7 years 1 month at 2%?

1. What is the interest of \$120 for 2 years 4 months 15 days, at 6%?

SOLUTION.

Interest of 1 year = .06×\$120, or \$7.20,
Interest of 2 years = 2×\$7.20, or \$14.40,
Interest of 1 month = \$7.20÷12, or \$.60,
Interest of 4.5 months = 4.5×\$.60, or \$2.70,
Interest of 2 years 4 months 15 days = \$14.40+\$2.70, or \$17.10.

NOTE.—Since 30 days make a month, any number of days are so many thirtieths of a month, which is reduced to 10ths by dividing by 3. Hence, the number of days divided by 3 gives tenths of a month. Thus 15 days = .5 month, 12 days = .4 month, etc. The months may be reduced to days, and the whole number of days be taken as a fraction of a year. Thus, 4 months 15 days = 135 days, or $\frac{135}{360}$ of a year = $\frac{3}{8}$ year. The total time would be $2\frac{3}{8}$ years.

Find the interest—

2. Of \$3000 for 3 years 6 months 24 days, at 6 per cent.
3. Of \$2500 for 4 years 4 months 18 days, at 6 per cent.
4. Of \$2000 for 3 years 7 months, at 6 per cent.
5. Of \$4000 for 2 years 5 months, at 6 per cent.
6. Of \$2000 for 1 year 9 months 18 days, at 6 per cent.
7. Of \$4000 for 2 years 1 month, at 12 per cent.
8. Of \$1200 for 5 years 10 months, at 12 per cent.
9. Of \$4800 for 3 years 7 months 6 days, at 5 per cent.
10. Of \$6000 for 5 years 3 months 18 days, at 8 per cent.
11. Of \$4500 for 2 years 3 months 9 days, at 6 per cent.
12. Of \$7500 for 2 years 2 months 12 days, at 5 per cent.
13. Of \$6500 for 1 year 6 months 12 days, at 10 per cent.
14. Of \$4500 for 1 year 6 months 20 days, at 9 per cent.
15. In interest what corresponds to the *base* in percentage?
16. In interest what corresponds to the *percentage?*
17. What element in interest not found in percentage?
18. How find the interest on any sum for 1 year?
19. How find the interest on any sum for 1 month?
20. How find the interest on any sum for 10 days?
21. How find the interest on any sum for 3 days?

1. What principal will gain \$210 in 2 years 4 months, at 6 per cent?

SOLUTION I.

Interest for $2\frac{1}{3}$ years, at 6 per cent will be \$.14 on each dollar,

The required principal will be \$1 taken as many times as \$.14 is contained times in the given interest, or \$1500.

SOLUTION II.

Interest for $2\frac{1}{3}$ years = \$210,
Interest for 1 year will be \$210 ÷ $2\frac{1}{3}$, or \$90,
6% of the principal = \$90,
1% of the principal = $\frac{1}{6}$ of \$90, or \$15,
100% of the principal = 100 × \$15, or \$1500.

SOLUTION III.

6% for $2\frac{1}{3}$ years = 14%,
14% of the principal = \$210,
1% of the principal = \$15,
100% of the principal = \$1500.

SOLUTION IV.

Principal : principal :: interest : interest,
x : \$1 :: \$210 : \$.14,
Solving, the required principal = \$1500.

Find the principal which will give an interest of—

2. \$210 in 7 yrs. at 5%.
3. \$240 in 8 yrs. at 6%.
4. \$300 in 3 yrs. at 8%.
5. \$700 in 7 yrs. at 5%.
6. \$600 in 8 yrs. at 5%.
7. \$400 in 3 yrs. at 6%.
8. \$720 in 1 yr. at 5%.
9. \$450 in 1 yr. at 6%.
10. \$440 in 4 yrs. at 5%.
11. \$500 in 6 yrs. 3 mos. at 4%.
12. \$640 in 5 yrs. 4 mos. at 6%.
13. \$660 in 3 yrs. 8 mos. at 3%.
14. \$300 in 5 yrs. at 6%.
15. \$420 in 5 yrs. at 7%.
16. \$840 in 5 yrs. at 8%.
17. \$720 in 4 yrs. at 10%.
18. \$600 in 4 yrs. at 6%.
19. \$540 in 5 yrs. at 8%.

20. How find the principal, if the rate and 1 year's interest are given?

The amount of a debt drawing interest is what it is worth at some future time. Its present value is the principal which, if put at interest, will amount to the debt when it becomes due. Sometimes a debt is contracted for a certain amount due at a fixed future time without interest, and a *Discount* is offered if paid before it becomes due. The discount is calculated at a given rate by the preceding solutions for finding the principal, when the amount, the time, and the rate per cent are given.

The *amount* of the debt, less the *discount* will give the present value of the debt, which is called the *present worth.*

The discount found by the above method is called *True Discount*, to distinguish it from *Bank Discount*, found by another method used by banks in buying notes.

1. What principal will amount to \$224 in 2 years, at 6 per cent?

SOLUTION I.

The principal = 100%,
Interest for 2 years = 12%,
The amount = 112% = \$224,
1 per cent = $\frac{1}{112}$ of \$224, or \$2.
100 per cent = 100×\$2, or \$200, the required principal.

SOLUTION II.

The interest in 2 years at 6 per cent = $\frac{12}{100}$, or $\frac{3}{25}$ of the principal,
The amount $\frac{25}{25}+\frac{3}{25}=\frac{28}{25}$ of the principal = \$224,
$\frac{1}{25}$ of the principal = $\frac{1}{28}$ of \$224, or \$8,
$\frac{25}{25}$ of the principal = 25×\$8, or \$200.

SOLUTION III.

Amount of \$1 in 2 years at 6 per cent = \$1.12,
Amount : amount :: principal : principal,
\$1.12 : \$224 :: \$1 : x,
Solving, the required principal = \$200.

2. Find the principal which will amount to \$284 in 7 years, at 6 per cent.

1. Find the rate at which \$300 will gain \$18 in 2 years.

SOLUTION I.

Interest for 2 years = \$18,
Interest for 1 year = 9,
\$9 ÷ \$300 = .03, or 3%.

SOLUTION II.

Interest at 1% = \$3 for 1 year,
Interest at 1% = \$6 for 2 years,
If \$6 = 1%, \$18 = 3%.

SOLUTION III.

Interest : interest :: rate : rate,
\$18 : 6 :: x% : 1%,
Solving the proportion, the rate = 3%.

Find the rate—

2. At which \$ 240 in 5 years will produce \$ 60 interest.
3. At which \$ 800 in 3 years will produce \$144 interest.
4. At which \$ 900 in 4 years will produce \$360 interest.
5. At which \$ 800 in 3 years will produce \$168 interest.
6. At which \$ 500 in 4 years will produce \$120 interest.
7. At which \$ 600 in $2\frac{1}{2}$ years will produce \$ 90 interest.
8. At which \$1000 in $1\frac{1}{2}$ years will produce \$ 90 interest.
9. At which \$ 450 in 10 years will produce \$450 interest.
10. At which \$ 60 in $3\frac{1}{3}$ years will produce \$ 18 interest.
11. At which \$4000 in 2 years will amount to \$4320.

At what rate—

12. Will \$1920 gain \$177.60 in 1 year 6 months 15 days?
13. Will \$1795 gain \$502.60 in 3 years 6 months?
14. Will \$2304 gain \$806.40 in 3 years 10 months 20 days?
15. Will \$2462.72 gain \$205.22$\frac{2}{3}$ in 8 years 4 months?
16. Will \$ 900 amount to \$1045.80 in 2 years 8 months 12 days?
17. A sum of money amounts to \$987 in 3 years 6 months, at 5%. In what time will it amount to \$956.20, at 6%?
18. Find the amount of \$907.20 from September 6, 1896, to June 14, 1897, at 9%.

1. In what time will \$400 gain \$84, at 6 per cent?

SOLUTION I.

\$24 equals the interest for 1 year,
\$ 1 equals the interest for $\frac{1}{24}$ of a year,
\$84 equals the interest for $\$84 \times \frac{1}{24}$, or $3\frac{1}{2}$ years.

SOLUTION II.

Interest : interest :: time : time.
\$84 : \$24 :: x : 1 year.
Solving, the required time is $3\frac{1}{2}$ years.

Find the time—

2. In which \$ 400 will produce \$ 100 at 5 %.
3. In which \$1000 will produce \$ 210 at 6 %.
4. In which \$2000 will produce \$ 450 at $4\frac{1}{2}$%.
5. In which \$5000 will produce \$1000 at 4 %.
6. In which \$1000 will produce \$ 227.50 at 7 %.
7. In which \$1500 will produce \$ 25 at $3\frac{1}{3}$%.
8. In which \$2000 will produce \$ 550 at 10 %.
9. In which \$3000 will produce \$ 150 at 12 %.
10. In which \$5000 will produce \$ 362.50 at 6 %.
11. In which \$2000 will produce \$ 480 at 8 %.
12. In which \$5000 will produce \$1283.33 at 10 %.
13. In which \$ 600 will amount to \$701.25 at 5%.
14. In which any principal will gain two times itself at 10%.
15. In which any principal will double itself at 6%.
16. In which any principal will double itself at 8%.

Find the time in which—

17. \$ 320 will gain \$ 26.56, at 6 %.
18. \$1800 will gain \$168, at $3\frac{1}{2}$%.
19. \$ 680 will gain \$102, at 6 %.
20. \$ 720 will gain \$ 80.40, at 5 %.
21. \$ 540 will gain \$ 67.20, at 4 %.
22. \$ 612 will gain \$ 95.20, at 7 %.
23. \$ 546 will gain \$143.78, at 8 %.
24. \$1680 will gain \$140, at $3\frac{1}{3}$%.
25. \$2100 will gain \$136.50, at 4 %.

To find the interest at 6 per cent of any given principal:

(1) For 2 months, remove the decimal point two places to the left.

(2) For 6 days, remove it three places to the left.

(3) Add multiples or parts of these results for any given time.

This is known as the "6 per cent method" of calculating interest.

1. What is the interest of \$250, at 6 per cent, for 2 months 6 days?

SOLUTION.

Interest for 2 months, (60 days) at 6 per cent=.01 of \$250=\$2.50,
Interest for 6 days ($\frac{1}{10}$ of 2 months) =.001 of \$250= .25,
Interest for 2 months 6 days, at 6 per cent \$2.75.

2. In like manner find the interest for 2 months 6 days, at 6 per cent, of \$224, \$348, \$642, \$124, \$760, \$875, \$1,580.

3. Find the interest of \$240 for 2 years 7 months 20 days, at 6 per cent.

SOLUTION.

Interest for 2 months = \$2.40; for 6 days = \$.24.
Interest for 2 years 6 months (30 months) = 15 × \$2.40 = \$36.00,
Interest for 1 month = $\frac{1}{2}$ of \$2.40 ..... = 1.20,
Interest for 18 days = 3 × \$ .24 ..... = .72,
Interest for 2 days = $\frac{1}{9}$ of \$ .24 ..... = .08,
Interest for 2 years 7 months 20 days ..... = \$38.00.

Find the interest at 6%—

4. Of \$412 for 5 mos.
5. Of \$640 for 5 mos.
6. Of \$720 for 5 mos.
7. Of \$420 for 2 mos. 20 das.
8. Of \$650 for 3 mos. 24 das.
9. Of \$650 for 1 yr. 8 mos.
10. Of \$108 for 2 yrs. 9 mos.
11. Of \$108 for 2 mos. 21 das.
12. Of \$214 for 3 mos. 12 das.
13. Of \$320 for 5 mos. 15 das.
14. Of \$512 for 7 mos. 9 das.
15. Of \$175 for 10 mos. 3 das.
16. Of \$175 for 5 mos. 9 das.
17. Of \$165 for 10 mos. 3 das.
18. Of \$165 for 5 mos. 9 das.
19. Of \$600 for 3 mos. 24 das.
20. Of \$600 for 3 yrs. 5 mos.
21. Of \$140 for 7 mos. 9 das.

1. Find the interest of \$125 for 1 year 4 months 18 days, at 7%. At 8%.

SOLUTION.

Interest for 2 months at 6% = \$1.25; for 6 days = \$ .125,
Interest for 16 months = 8 × \$1.25 ..... = 10.00,
Interest for 18 days = 3 × 1.25 ..... = .375.

Interest at 6 per cent for 1 year 4 months 18 days = \$10.375,
Interest at 1 per cent is $\frac{1}{6}$ of \$10.375 ..... = 1.729,*
Interest at 7 per cent (by adding) ..... = \$12.104.

To find the interest at 8 per cent—
Interest at 6 per cent ..... = \$10.375,
Interest at 2 per cent ($\frac{1}{3}$ of 6%) = $\frac{1}{3}$ of \$10.375... = 3.458,*
Interest for 8 per cent (by adding) ..... = \$13.833.*

Find the interest—

2. Of \$1200, at 12%, for 22 months 12 days.
3. Of \$2000, at 6%, for 2 years 6 months 15 days.
4. Of \$2000, at 5%, for 2 years 8 months 18 days.
5. Of \$7200, at 7%, for 2 years 8 months 20 days.
6. Of \$2400, at 8%, for 1 year 4 months 10 days.
7. Of \$1800, at 9%, for 1 year 2 months 10 days.
8. Of \$1600, at 10%, for 3 years 4 months 24 days.
9. Of \$2650, at 11%, for 2 years 1 month 6 days.
10. Of \$2400, at 12%, for 11 years 9 months 6 days.
11. Of \$1800, at 5%, for 4 years 4 months 15 days.
12. Of \$7200, at 8%, for 6 years 3 months 10 days.
13. Of \$1200, at 6%, for 3 years 9 months 24 days.
14. Of \$8400, at 10%, for 1 year 2 months 2 days.
15. Of \$8100, at 9%, for 3 years 8 months 12 days.
16. Of \$4320, at 8%, for 3 years 9 months 15 days.
17. Of \$3600, at 12%, for 2 years 6 months 10 days.
18. Of \$9000, at 7%, for 2 years 8 months 15 days.

*In calculating interest, one half cent or more is considered as one cent; less than one half cent is dropped.

For short time the most convenient method of computing interest at 6% is to reduce the time to days, and to take .01 of the principal for 60 days' interest, and such multiples or parts as the time requires.

1. Find the interest of \$1,260 for 2 months 20 days, at 6 per cent.

SOLUTION.

Time = 80 days.
Interest for 60 days (2 months) = \$12.60,
Interest for 20 days = 4.20,
Interest for 2 months 20 days = \$16.80.

2. Find the interest of \$1,200, at 8%, for 90 days.

SOLUTION.

$\frac{\$1200\times.08\times90}{360} = \$24.$

\$1200×.08 = interest for what time? Why divide by 360? Why multiply by 90? Use cancellation.

By any preceding method find the interest—

3. Of \$ 490, for 6 months 18 days, at 3½%.
4. Of \$1690, for 1 year 2 months 13 days, at 4 %.
5. Of \$4960, for 2 years 5 months 7 days, at 4½%.
6. Of \$1920, for 3 years 7 months 9 days, at 5 %.
7. Of \$1699, for 4 years 4 months 12 days, at 6 %.
8. Of \$1423, for 3 years 5 months 10 days, at 6 %.
9. Of \$ 360, for 1 year 6 months 15 days, at 7 %.
10. Of \$ 950, for 1 year 11 months 6 days, at 7½%.
11. Of \$4310, for 2 years 7 months 22 days, at 8 %.
12. Of \$1500, for 2 years 6 months 27 days, at 9 %.
13. Of \$8000, for 2 months 15 days, at 7.3%.
14. Of \$9390, for 1 month 18 days, at 10%.
15. Of \$ 959.70, for 106 days, at 6%.
16. Of \$1896.50, for 89 days, at 5%.
17. Of \$ 421.60, for 45 days, at 4%.
18. Of \$ 435.60, for 110 days, at 6%.

INTEREST AT 6 PER CENT FOR MULTIPLES AND DIVISORS OF 200 MONTHS.

At 6 per cent, interest for 2 months is 1 per cent of the principal; interest for 20 months is 10 per cent, or $\frac{1}{10}$, of the principal; and interest for 200 months is 100 per cent of the principal, or the principal itself. Hence—

Interest for $\frac{1}{2}$ of 200 months, or 100 months, equals $\frac{1}{2}$ of the principal; interest for $\frac{1}{3}$ of 200 months, or 66 months 20 days, equals $\frac{1}{3}$ of the principal, etc.; and interest for $\frac{1}{2}$ of 20 months, or 10 months, $= \frac{1}{2}$ of $\frac{1}{10}$ of the principal; interest for $\frac{1}{3}$ of 20 months, or 6 months 20 days, equals $\frac{1}{3}$ of $\frac{1}{10}$ of the principal, etc.

Apply these suggestions in the solution of the following problems, by using multiples or parts of 2 months, 20 months, or 200 months.

1. What is the interest of \$547.98 for 33 months 10 days?

Suggestion: 33 months 10 days $= 33\frac{1}{3}$ months $= \frac{1}{6}$ of 200 months.

What is the interest—

2. Of \$84.28 for 200 mo.?
3. Of \$60.72 for 100 mo.?
4. Of \$36.60 for $66\frac{2}{3}$ mo.?
5. Of \$24.33 for 50 mo.?
6. Of \$21.75 for 40 mo.?
7. Of \$18.72 for $33\frac{1}{3}$ mo.?
8. Of \$29.75 for 25 mo.?
9. Of \$54.80 for 20 mo.?
10. Of \$13.74 for $16\frac{2}{3}$ mo.?
11. Of \$75.30 for $13\frac{1}{3}$ mo.?
12. Of \$64.16 for $12\frac{1}{2}$ mo.?
13. Of \$4.67 for 16 yr. 8 mo.?
14. Of \$429 for 8 yr. 4 mo.?
15. Of \$725 for 2 yr. 1 mo.?
16. Of \$768 for 4 yr. 2 mo.?
17. Of \$6.54 for 1 yr. $1\frac{1}{3}$ mo.?
18. Of \$87.32 for 1 yr. 8 mo.?
19. Of \$25.30 for 3 yr. 4 mo.?
20. Of \$57.84 for 20 mo.?
21. Of \$684 for 6 mo. 20 da.?
22. Of \$2872 for 5 mo.?
23. Of \$69.80 for 4 mo.?
24. Of \$ 3.24 for 3 mo. 10 da.?
25. Of \$578 for 2 mo. 15 da.?
26. Of \$45.15 for 4 da.?
27. Of \$357 for 20 da.?
28. Of \$ 41 for 15 da.?
29. Of \$837 for 12 da.?
30. Of \$582 for 10 da.?
31. Of \$ 15 for 4 da.?
32. Of \$896 for 3 da.?
33. Of \$555 for 2 da.?
34. Of \$489 for 1 da.?
35. Of \$954 for 4 mo.?

The United States reckons *exact interest* for time less than a year, by counting the exact number of days, and considering 365 days a year. The method is indicated by the solution of the next problem.

1. Find the exact interest of a U. S. $1,000 bond from December 1, 1898, to March 15, 1,899, at 5%.

SOLUTION.

Days in December, 30,
Days in January, 31,
Days in February, 28,
Days in March, 15,
Total number days, 104.

$$\frac{\$1{,}000 \times .05 \times 104}{365} = \$14.246, \text{ the interest.}$$

Find the exact interest of—

2. Of $512.30 from May 17 to July 31, at 6%.
3. Of $286.90 from September 18 to December 13, at 7%.
4. Of $374.88 from May 25 to August 29, at 7.3%.
5. Of $3000 U. S. bonds from February 15 to June 5, at 7.3%.
6. Of $451 from March 16, 1895, to January 13, 1896, at 5%.

Find the common interest and the exact interest—

7. Of $1221.50, at 12%, from July 1, 1903, to Oct. 1, 1903.
8. Of $ 886.40, at 10%, from July 1, 1903, to Nov. 25, 1903.
9. Of $ 555.50, at 7%, from July 11, 1903, to Dec. 25, 1903.
10. Of $ 530.30, at 8%, from July 1, 1903, to Jan. 1, 1904.
11. Of $ 700, at 5%, from July 1, 1903, to April 1, 1904.
12. Of $1150, at 6%, from July 1, 1903, to May 30, 1904.

What principal will amount—

13. To $ 542.10 in 2 years 6 months 12 days, at 6%?
14. To $1248.10 in 1 year 6 months 21 days, at 7%?
15. To 1893\frac{1}{8}$ in 1 year 2 months 15 days, at 8%?
16. To $ 326.72 in 3 years 2 months 18 days, at 9%?

Find the amount—

1. Of \$1200, at 9%, for 9 months 4 days.
2. Of \$1680, at 6%, for 1 year 8 months 15 days.
3. Of \$ 960, at 7%, for 2 years 4 months 24 days.
4. Of \$2496, at 8%, for 2 years 5 months 9 days.
5. Of \$1080, at 4%, for 2 years 9 months 21 days.
6. Of \$1100, at 12%, for 3 years 6 months 18 days.
7. Of \$ 900, at 11%, for 4 years 8 months 18 days.
8. Of \$ 796, at 5%, for 4 years 10 months 6 days.
9. Of \$1000, at 7%, for 3 years 9 months 27 days.
10. Of \$ 86, at 9%, for 4 years 4 months 15 days.
11. Of \$1126, at 6%, for 20 years 5 months 1 day.
12. Of \$1295, at 8%, for 3 years 2 months 21 days.

13. I refused \$1,200 cash instead of a note for \$1,280 due in a year without interest. If interest is 8%, what did I gain or lose?

14. A real estate agent bought property for \$7,572, and sold it for \$10,000, after keeping it for 1 year 6 months. What would he have gained or lost, by loaning his money at 8% instead of investing in the property?

15. How find the interest for more than 1 year?
16. How find the interest for 1 month? For 5 months?
17. How find the interest for 1 day? For 9 days?
18. At the rate of 6 per cent, what is the rate for 1 month?
19. At the rate of 6 per cent, what is the rate for 2 months?
20. How indicate 2 months' interest at 6 per cent?
21. How indicate 6 days' interest at 6 per cent?
22. Given 2 months' interest, how find interest for any number of months?
23. Given 6 days' interest, how find interest for any number of days?
24. How many days are usually reckoned for a month?
25. What is meant by *exact interest*?
26. Write the rule for computing interest, common method.
27. Write the rule for computing interest, 6 per cent method.
28. Write the rule for computing the exact interest.

COMPOUND PROPORTION.

A COMPOUND PROPORTION is the comparison of the terms of two equal ratios, when one or both are compound.

A COMPOUND RATIO is reduced to a simple one, by multiplying all the antecedents together for a new antecedent, and all the consequents together for a new consequent.

1. If 8 men in 12 days can build 80 rods of wall, how much will 6 men build in 18 days?

SOLUTION.

$$\left.\begin{matrix}8 \text{ men}\\ 12 \text{ days}\end{matrix}\right\} : \left.\begin{matrix}6 \text{ men}\\ 18 \text{ days}\end{matrix}\right\} :: 80 \text{ rds.} : x \text{ rds.} \quad x = \frac{18\times 6\times 80}{12\times 8} \text{ or } 90 \text{ rds.}$$

2. If 20 men can perform a piece of work in 12 days, working 9 hours a day, how many men will accomplish the same work in 6 days, working 10 hours a day?

SOLUTION.

$$\left.\begin{matrix}6\\ 10\end{matrix}\right\} : \left.\begin{matrix}12\\ 9\end{matrix}\right\} :: 20 : x. \quad x = \frac{12\times 9\times 20}{6\times 10} \text{ or } 36 \text{ men.}$$

Since 20 men require 108 hours to do the work, more men will be required to do the same work in 60 hours. The terms are therefore *inversely* proportional.

The relations of the terms of a proportion may also be regarded in many problems as a comparison of two *causes* and their corresponding *effects*, expressed as follows—

1st cause : 2d cause :: 1st effect : 2d effect.

3. If 6 men in 4 days, working 10 hours a day, can reap 16 acres, in how many days can 10 men, working 12 hours a day, reap 24 acres?

SOLUTION.

1st Cause. 2d Cause. 1st Effect. 2d Effect.

$$\left.\begin{matrix}6 \text{ men}\\ 4 \text{ days}\\ 10 \text{ hrs.}\end{matrix}\right\} : \left.\begin{matrix}10 \text{ men}\\ x \text{ days}\\ 12 \text{ hrs.}\end{matrix}\right\} :: 16 \text{ acres} : 24 \text{ acres.}$$

$$x = \frac{6\times 4\times 10\times 24}{10\times 12\times 16} \text{ or } 3 \text{ days.}$$

1. If 6 men can make 216 pairs of shoes in 2 days, how many pairs can 2 men make in 6 days?

2. If 18 men can earn $324 in 36 days, how many dollars can 42 men earn in 27 days?

3. If $200 will earn $140 interest in 7 years, in what time will $500 earn $360 at the same rate?

4. If $400 will, in 9 months, gain $21, when the rate of interest is 7% per annum, how much will $360 gain in 8 months, if the rate per cent is 6?

5. If $400 require 9 months to gain $21 when the rate per cent is 7, how long a time will $360 require to gain $14.40, if the rate per cent is 6?

6. If $400, to gain $21, in 9 months, require a rate of 7 per cent, what must be the rate per cent for $360 to gain $14.40 in 8 months?

7. If it require $400 to gain $21 in 9 months when the rate per cent is 7, how much will be required to gain $14.40 in 8 months, when the rate per cent is 6?

8. If a bar of iron 5 feet long, $2\frac{1}{2}$ inches wide and $1\frac{3}{4}$ inches thick, weigh 45 pounds, how much will a bar of the same metal weigh that is 7 feet long, 3 inches wide, and $2\frac{1}{2}$ inches thick?

9. If $800 gain $20 in 6 months, how long will it take $600 to gain $100 at the same rate of interest?

10. If $1,000 in 3 years 6 months, at 6 per cent, gain $210 interest, how much will $1,200 gain in 4 years 9 months, at 8 per cent?

11. If $500 gain $35 in 12 months, what will $450 gain in 8 months at the same rate?

12. If 32 men can dig a trench 120 rods long, 15 feet wide, and 12 feet deep, how many men are required to dig a trench 360 rods long, 9 feet wide, and 10 feet deep.

NOTE.—All the dimensions of a surface or a solid belong in the same term of a proportion; so that, having decided where one dimension belongs, the others can be written under this at once.

13. Write an original problem in compound proportion.

1. If 120 men in 3 days, of 12 hours each, can dig a trench 30 yards long, 2 feet broad, and 4 feet deep, how many men would be required to dig a trench 50 yards long, 6 feet deep, and $1\frac{1}{2}$ yards broad, in 9 days of 15 hours each?

2. If a stream of water running into a pond of 175 acres raises it 10 inches in 15 hours, how much would a pond of 80 acres be raised by the same stream in 9 hours?

3. A person having a journey of 500 miles to perforn, walks 200 miles in 8 days, walking 12 hours a day. In how many days, walking 10 hours a day, will he complete the remainder of the journey?

4. If 6 men can mow 30 acres of grass in 5 days, working 8 hours each day, how many acres can 4 men mow in 6 days, of 10 hours each?

SOLUTION—BY ANALYSIS.

6 men in 5 days working 8 hours a day can mow 30 acres,

1 man in 5 days working 8 hours a day can mow $\frac{1}{6}$ of 30 acres, or 5 acres,

1 man in 1 day working 8 hours a day can mow $\frac{1}{5}$ of 5 acres, or 1 acre,

1 man in 1 day working 1 hour a day can mow $\frac{1}{8}$ of an acre,

4 men in 1 day working 1 hour a day can mow $4 \times \frac{1}{8}$ of 1 acre or $\frac{1}{2}$ acre,

4 men in 9 days working 1 hour a day can mow $9 \times \frac{1}{2}$ of 1 acre or $\frac{9}{2}$ acre,

4 men in 9 days working 10 hours a day can mow $10 \times \frac{9}{2}$ acres, or 45 acres.

5. If 5 compositors in 16 days, working 8 hours a day, can compose 20 sheets of 24 pages each, 50 lines in a page, and 40 letters in a line, in how many days, working 9 hours a day, can 10 compositors compose 40 sheets of 16 pages in a sheet, 60 lines in a page, and 56 letters in a line?

NOTE.—Solve both by cause and effect and by analysis.

6. When it is 6 A. M. at Boston, what time is it at Portland, Oregon? At Chicago? At Denver?

PARTNERSHIP WITH TIME involves two elements—the amount of capital, and the time it is invested.

1. A, B, and C engage in trade. A puts in \$400 for 2 months, B \$300 for 4 months, and C \$500 for 3 months. They gain \$350. Find each man's share of the gain.

SOLUTION.

B's \$300 for 4 months = \$1200 for 1 month,
A's \$400 for 2 months = 800 for 1 month,
C's \$500 for 3 months = 1500 for 1 month,
The entire stock is the same as \$3500 for 1 month,
Therefore, A must have $\frac{800}{3500}$, or $\frac{8}{35}$ of \$350 = \$ 80,
B must have $\frac{1200}{3500}$, or $\frac{12}{35}$ of \$350 = \$120,
C must have $\frac{1500}{3500}$, or $\frac{3}{7}$ of \$350 = \$150.

2. A, B, and C had a joint stock of \$4,800. A's part was \$1,500, and continued in trade 4 months; B's was \$1,700, and continued 8 months; the remainder was C's, and continued in trade throughout the year. They lost \$280. What was the loss of each?

3. A, B, and C were in partnership. A had in the business \$5,000 for 4 months, B \$4,000 for 6 months, and C \$6,000 for 7 months. The profits were \$1,290. What was the profit of each?

4. Three persons formed a partnership. B put in \$250 for 6 months, C \$275 for 8 months, and D \$450 for 4 months. They gained \$825. What was each man's share of the gain?

5. Two merchants formed a partnership for 18 months. A at first put in \$1,000, and at the end of 8 months he put in \$600 more. B at first put in \$1,500, but at the end of 4 months he drew out \$300. At the expiration of the time they had gained \$697.32. What was each man's share of the gain?

6. A, B, and C entered into partnership. A put in \$1,000 for 18 months, B \$760 for 13 months, and C \$540 for 9 months. They lost \$1,037. What was each man's loss?

7. A, B, and C formed a co-partnership. A furnished $\frac{1}{5}$ of the capital for 6 months, B $\frac{1}{8}$ of the capital for 10 months, C the remainder for 12 months. The whole gain was \$3,120. What was the gain of each?

1. A, B and C entered into partnership January 1. A put in $500, B $700, and C $100. Four months later they took in D, who put in $400. On settling, at the close of one year, they found they had made $800. What was each man's share of the gain?

2. A and B enter into partnership. A puts in $600 January 1; B puts in nothing until April 1. How much must he then put in, in order that the partners may share the gain equally at the end of the year?

3. Messrs. White, Jones and Saylor formed a partnership in the wholesale grocery business. White put in $6,000 and added $3,000 more at the end of the first year; Jones put in $10,000, and at the end of 8 months withdrew $2,000; Saylor invested $8,000, and at the end of a year and a half withdrew $1,600. At the close of two years they had gained $17,112. How should it be divided?

4. A, B, and C began business January 1 with $1,300, furnished by A; April 1, B put in $1,000; July 1, C put in $900. The profit for the year was $750. Find each man's share?

5. A, B, and C were interested in a coal mine, and cleared the first year $6,570. A had had $20,000 invested for 9 months, when he withdrew half of that sum; B put in $24,000 2 months after the partnership was formed, and C put in $24,000 5 months before the year closed. Divide the profit.

6. January 1, A and B formed a partnership, A putting in $8,000 and B $2,000. April 1 A put in $4,000 more, and June 1 B put in $2,000 more. October 1 each took out $2,600. At the end of the year they had gained $5,488. What was the gain of each?

7. On the first day of January, A began business with $650; on the first day of April following he took B into partnership, with $500; on the first day of next July, they took in C, with $450. At the end of the year their gain was $375. What share of the gain had each?

8. At 8 A. M. at New York, what is the time at San Francisco?

SOLIDS AND CAPACITIES.

The volume of a prism or a cylinder 5 feet in length is 5 times as much as 1 foot in length of the same prism or cylinder. Hence, the volume of a prism or cylinder is equal to the product of the area of its base multiplied by its length.

CAPACITY OF BINS.

One bushel = 2,150.4 cubic inches. One cubic foot = 1,728 cubic inches. A bushel (2,150.4÷1,728) is about 1.25 cubic feet. (Divide for exact result.) This is not *exact*, but is so nearly exact that it is used to estimate the contents of a bin in bushels of grain.

1. How many bushels of oats in a wagon box 10 feet long, 3 feet wide and 18 inches high?

NOTE.—How much grain to each inch in height? After the contents of a box 10 feet long are known, how find the contents of one 11 feet long? 12 feet long? Inside measurement must be reckoned.

2. How many bushels of grain in a wagon box 12 feet long, 3 feet wide, and 2 feet high?

3. How many bushels of grain in a box 10 feet long, 3 feet wide, and 22 inches high?

4. How many bushels of grain in a bin 3 feet wide, 6 feet high, and 10 feet long?

5. How many bushels of grain in a bin 3 feet wide, 6 feet high, and 15 feet long?

6. How many bushels of grain in a bin 4 feet wide, 6 feet high, and 10 feet long?

7. How many bushels of grain in a bin 4 feet wide, 6 feet high, and 15 feet long?

8. How many bushels of grain in a bin 6 feet wide, 5 feet high, and 10 feet long?

9. How long must be a bin 3 feet wide, and 4 feet high, to contain 144 bushels of grain?

10. How long must be a bin 5 feet wide, and 5 feet high, to contain 400 bushels of wheat?

Large fruits and vegetables, such as apples, potatoes, etc., are measured by "heaped measure," which makes each bushel about ¼ more than a level or "stroked" bushel. 1¼ times 2,150.4 cubic inches are 2,688 cubic inches, or (2,688÷1,728) about *1.5 cubic feet*. (Divide for exact result.)

1. How many bushels of potatoes will fill a box 12 feet long, 3 feet wide, and 2 feet high?

2. How many cubic feet required for storing 1,600 bushels of apples?

3. A bin is 5 feet wide by 10 feet long. How high must it be to contain 200 bushels of potatoes?

Where corn is raised to any extent, it is stored in a bin or crib before it is shelled from the cob. Shelled corn is estimated as other grain, but *ear corn* is estimated at 1 bushel for *2.5 cubic feet*.

4. A corn crib is 6 feet wide, 10 feet high, and 20 feet long. How many bushels of unshelled corn will it contain?

5. How much space is required for 600 bushel of unshelled corn?

6. A crib is 12½ feet wide, and 8 feet high. How long must it be to contain 1,000 bushels of unshelled corn?

CAPACITY OF TANKS AND CISTERNS.

A gallon contains 231 cubic inches. A cubic foot contains (1,728÷231) about *7.5 gallons*. (Divide for exact result.) If estimated in barrels, 31½ gallons are considered a barrel, and a barrel requires (31½×231÷1,728) about *4.2 cubic feet*.

7. Find the number of gallons in a tank 4 feet long, 18 inches high, and 2 feet wide.

8. A tank is 4 feet wide, and 5 feet long. How high must it be to contain 750 gallons?

9. How many gallons in a rectangular tank 2 feet long, 1 foot high, and 16 inches wide?

10. A circular cistern is 6 feet in diameter. How many gallons of water in the cistern, if the water is 6 inches deep? 3 feet deep? 5½ feet deep?

BRICK AND STONE WORK.

BRICK WORK is estimated by the *thousand bricks*, and 22 bricks 8 in.x4 in.x2 in. are required for each cubic foot. *Stone Work* is estimated in perches, a *perch* containing *24.75 cu. feet.* In estimating stone and brick work, the outside measurement is taken around all corners. Deductions are sometimes made for very large openings. Methods of such extensions vary in different localities.

1. Excavation for a cellar was 20 feet long, 16 feet wide, and 8 feet deep. How many perch of stone required to wall it with a wall 2 feet thick? (A part of a perch is reckoned as a whole one.)

2. A wall 2 feet thick and 9 feet high was built under a house 25 feet by $37\frac{1}{2}$ feet. Find the cost at $\$1.12\frac{1}{2}$ a perch.

3. Find the cost of bricks for the walls of a room 20 feet wide, 30 feet long, 14 feet high, the wall to be 18 inches thick, at \$5.50 per M.

Sometimes the bricks in a wall are found by estimating the bricks required for a square foot of wall surface for *each brick in thickness* of the wall. Thus,

8 in.$+\frac{1}{4}$ in. for mortar $= 8\frac{1}{4}$ inches, length of each brick.

2 in.$+\frac{1}{4}$ in. for mortar $= 2\frac{1}{4}$ inches, thickness of each brick.

$8\frac{1}{4}\times 2\frac{1}{4} = 18\frac{9}{16}$ sq. inches used by each brick.

$144\div 18\frac{9}{16} = 7\frac{8}{33}\frac{8}{3}$ bricks required for each *square foot of surface.*

A wall two bricks thick will require $2\text{x}7\frac{8}{33}\frac{8}{3}$ bricks for each square foot of surface, etc.

4. How many bricks required to build the walls given in problem 3, above, the walls being 4 bricks thick?

5. How many cubic yards of earth were removed in digging a cellar 28 ft. 9 in long, 22 ft. 8 in. wide, and 7 ft. 6 in. deep?

6. What are the solid contents of a block of wood $7\frac{3}{4}$ yards long, $\frac{3}{4}$ yard wide, $\frac{2}{3}$ yard thick?

7. Two of the dimensions of a stone column which contains 196.8 cu. ft., are 12.3 ft. and 4 ft. What is the shape of one end of the column?

1. Find the cords in three piles of wood—
The first being 8 ft. long, 4 ft. wide, and 3 ft. high.
The second being 12 ft. long, 4 ft. wide, and 2 ft. high.
The third being 10 ft. long, 4 ft. wide, and 3 ft. high.

2. How much wood in a pile 5 meters long, 1 meter wide, and 2 meters high?

3. How many decasteres in a pile of wood 10 meters long, 1 meter wide, and 3 meters high?

4. How high is a pile of wood which contains 15 cd. ft., if it is 10 ft. long, and 4 ft wide?

5. What will be the cost in United States money, of a cylinder of oil whose diameter is 3 feet, and whose length is 4 feet, if the oil is worth a franc per gallon?

6. How high must a bin 12 feet long, and 4 feet wide be, to hold 300 bushels of potatoes?

7. A farmer's crop of onions just filled a bin 5 ft. long, 3 ft. wide, and 4 ft. 6 in. high. How many bushels had he?

8. How many tons of wheat can be put in a bin 10 ft. long, 6 ft. 9 in. high, and 2 yds. wide?

9. A garden, 6 rods wide and 12 rods long, has a wall 2 feet thick, and 4 feet high around it, outside of the line. What was the cost of the wall at $3.33\frac{1}{3}$ a perch?

10. What will be the cost of digging a ditch around the garden above mentioned, within and adjacent to the wall, 3 feet wide and $2\frac{1}{2}$ feet deep, at 15 cents a cubic yard?

11. What would be the cost of walling the garden above mentioned, the central line of the wall to be on the boundary, at $\$1.62\frac{1}{2}$ a perch?

12. Prove that a standard bushel measure, $18\frac{1}{2}$ inches in diameter, and 8 inches deep, contains 2150.4 cubic inches.

13. A bin of wheat is 12 feet long, 8 feet wide, and 6 feet deep. It was bought by measure at 75 cents per bushel, and sold by weight, 28,368 lbs., at 80¢ per bushel. What was the gain?

14. Each pupil will present an original problem in solid mensuration.

What is the per cent gain, if the profit is—

1. $\frac{1}{8}$ the selling price?
2. $\frac{1}{4}$ the selling price?
3. $\frac{1}{5}$ the selling price?
4. $\frac{1}{6}$ the selling price?
5. $\frac{2}{3}$ the selling price?
6. $\frac{3}{4}$ the selling price?
7. $\frac{3}{5}$ the selling price?
8. $\frac{1}{7}$ the selling price?
9. $\frac{1}{9}$ the selling price?
10. $\frac{2}{7}$ the selling price?
11. $\frac{3}{7}$ the selling price?
12. $\frac{3}{8}$ the selling price?
13. $\frac{5}{8}$ the selling price?
14. $\frac{1}{3}$ the selling price?
15. $\frac{4}{5}$ the selling price?
16. $\frac{2}{5}$ the selling price?

What per cent is gained when the cost is—

17. $\frac{2}{3}$ of the selling price?
18. $\frac{3}{5}$ of the selling price?
19. $\frac{5}{6}$ of the selling price?
20. $\frac{4}{5}$ of the selling price?
21. $\frac{4}{7}$ of the selling price?
22. $\frac{5}{7}$ of the selling price?
23. $\frac{3}{8}$ of the selling price?
24. $\frac{5}{8}$ of the selling price?

What is the gain or loss per cent if you buy for—

25. \$.08 and sell for \$.12?
26. \$.12 and sell for \$.08?
27. \$.12 and sell for \$.16?
28. \$.18 and sell for \$.30?
29. \$.30 and sell for \$.24?
30. \$.30 and sell for \$.12?
31. \$.30 and sell for \$.36?
32. \$.24 and sell for \$.20?
33. \$.24 and sell for \$.32?
34. \$.42 and sell for \$.63?
35. \$.42 and sell for \$.70?
36. \$.56 and sell for \$.42?
37. \$.64 and sell for \$ .80?
38. \$.64 and sell for \$ .72?
39. \$.70 and sell for \$ .77?
40. \$.70 and sell for \$ .56?
41. \$.70 and sell for \$ .84?
42. \$.90 and sell for \$1.35?
43. \$.90 and sell for \$4.50?
44. \$$\frac{1}{3}$ and sell for \$$\frac{1}{2}$?
45. \$$\frac{1}{8}$ and sell for \$$\frac{1}{4}$?
46. \$$\frac{1}{8}$ and sell for \$$\frac{1}{5}$?
47. 50 cts. and sell for \$$\frac{5}{8}$?
48. \$$\frac{7}{8}$ and sell for 75 cts.?

What per cent of an acre is a lot—

49. 16 rods by 5 rods?
50. 16 rods by 2 rods?
51. 16 rods by 4 rods?
52. 16 rods by 9 rods?
53. 4 rods by 5 rods?
54. 8 rods by 10 rods?
55. 8 rods by 5 rods?
56. 12 rods by 10 rods?

57. A man owns 6% stock bought at 90, and 8% stock bought at 120. If he sells the stock at par, at the close of one year, what has he gained or lost on a share?

To be solved mentally from dictation—

1. Square 4, add 4, multiply by 2, add 9, square root,* less 3, multiply by 10, divide by 5, square, equals what?

2. Square 5, add 11, divide by 4, square root, multiply by 8, add 1, square root, cube, subtract 4, square root, equals what?

3. Cube 3, add 9, divide by 9, square, add 9, square root, add 4, square root, cube, subtract 2, square root, equals what?

4. Square 5, multiply by 3, subtract 3, divide by 8, square root, multiply by 8, add 1, square root, multiply by 2, square, divide by 2, equals what?

5. Give an original exercise similar to those above.

6. The difference between two numbers is 7, and their sum is 37. Find the numbers.

7. If .62½ of a bin contains 45 bushels of wheat, how many bushels in 87½ per cent of the bin?

8. If ¾ the square of a number is 27, what is the number?

9. What is the difference between ½ of the square of 16 and the square of ½ of 16?

10. The square of ⅔ a number is 100. Find the number.

11. The square of a number is 225. Find the square of ½ the number.

12. The cube of twice a number is how many times the cube of the number?

13. The square of three times a number is how many times the square of the number?

14. What per cent of the square of a number is the square of ¼ the number?

15. The square of 2 times a number is 32 more than 2 times the square of the number. What is the number?

16. The difference between the square of 3 times a number and 3 times the square of a number is 54. Find the number.

17. The difference between the square of 4 times a number and 4 times the square of a number is 300. Find the number.

*The *square root* of a number is a factor used twice to produce the number. See page 44.

1. Find the interest of \$80 for 72 days, at 5 per cent.

SOLUTION.

Interest for 60 days at 6 per cent = \$.80,
Interest for 12 days at 6 per cent = .16,
Interest for 72 days at 6 per cent = \$.96,
Interest at 1 per cent = $\frac{1}{6}$ of \$.96 = .16,
Interest at 5 per cent (subtracting) = \$.80.

Find the interest of—

2. \$100 for 1 yr. 8 mos. at 6.
3. \$200 for 2 yrs. 9 mos. at 6.
4. \$300 for 1 yr. 5 mos. at 6.
5. \$400 for 3 yrs. 2 mos. at 6.
6. \$500 for 2 yrs. 6 mos. at 6.
7. \$600 for 2 yrs. 7 mos. at 6.
8. \$700 for 3 yrs. 3 mos. at 6.
9. \$600 for 90 days at 7.
10. \$540 for 60 days at 5.
11. \$540 for 48 days at 5.
12. \$350 for 120 days at 9.
13. \$700 for 72 days at 4.
14. \$288 for 60 days at 10.
15. \$540 for 60 days at 3.
16. Find the amount of \$ 500 for 4 years, at 5 per cent.
17. Find the amount of \$ 500 for 7 years, at 10 per cent.
18. Find the amount of \$ 300 for 5 years, at 7 per cent.
19. Find the amount of \$2300 for 2 years 8 mos., at 8 per cent.
20. Find the amount of \$2200 for 7 years 6 mos., at 6 per cent.
21. Find the amount of \$3200 for 3 years 9 mos., at 8 per cent.
22. Find the amount of \$1600 for 8 years 10 mos., at 6 per cent.
23. Find the amount of \$1400 for 8 years 4 mos., at 6 per cent.
24. Find the amount of \$1400 for 2 yrs. 6 mos. 20 das. at 9 per cent.
25. Find the amount of \$1500 for 8 years, at $5\frac{1}{2}$ per cent.
26. Find the amount of \$1406 for 6 years, at $4\frac{1}{2}$ per cent.
27. Find the amount of \$1100 for $4\frac{1}{2}$ years, at 10 per cent.
28. In what time will \$125 earn \$45, at 6 per cent.
29. In what time will \$ 80 earn \$10, at 5 per cent.
30. In what time will \$150 earn \$31, at 5 per cent.
31. In what time will \$100 earn \$18, at 6 per cent.
32. In what time will \$500 earn \$70, at 4 per cent.
33. In what time will \$120 earn \$ 9, at 3 per cent.
34. In what time will \$400 earn \$91, at 7 per cent.

Find the principal—

1. That will earn $224 in 2 years, at 4 per cent.
2. That will earn $420 in 6 years, at 5 per cent.
3. That will earn $120 in 8 years, at 6 per cent.
4. That will earn $420 in 7 years, at 4 per cent.
5. That will earn $480 in 8 years, at 5 per cent.
6. That will earn $192 in 5 years, at 8 per cent.
7. That will earn $ 16.80 in 7 months, at 6 per cent.
8. That will earn $ 16.80 in 6 months, at 7 per cent.
9. That will earn $144 in 2 years 8 mths., at 6 per cent.
10. That will earn $180 in 3 years 3 mths., at 6 per cent.
11. That will earn $ 65 in 10 years 10 mths., at 6 per cent.

12. What is the present worth of $104, due in 5 years, discounted at 6 per cent?

13. What is the face of a note which will amount to $5,200 in 3 years 4 months, at 9 per cent?

14. Find the present worth and discount of $340, due in 7 years, discounted at 10 per cent.

15. Find the present worth and discount of $290, due in 5 years, discounted at 9 per cent.

16. Find the present worth and discount of $700, due in 8 years, discounted at 5 per cent.

17. Find the present worth and discount of $290, due in 7 years 6 months, discounted at 6 per cent.

18. A's money is 4 times B's, and the sum of what they both have, in 3 years 4 months, at 6 per cent, will amount to $1,800. Find what each has.

Find the per cent realized—

19. By investing in 4 per cent bonds at 80.
20. By investing in 5 per cent bonds at 125.
21. By investing in 4½ per cent bonds at 90.
22. By investing in 5½ per cent bonds at 110.

23. An agent lost 40% on a wagon by selling it for $60. For what would he have sold it to gain 40%?

1. Explain why the square of $\frac{2}{3}$ is greater than its cube.

2. The sum of the ages of husband and wife was $87\frac{1}{2}$ years, and the difference was $12\frac{1}{2}$ years. Find their ages.

3. Market reports telegraphed from London to New York, 74° W., at 3:15 P. M., should reach New York at what hour, accounting for 10 minutes delay in sending?

4. If 8 yards of muslin, $1\frac{1}{4}$ yards wide, cost \$1.25, what will be the cost of 10 yards, $1\frac{1}{8}$ yards wide, at the same rate?

5. How many gallons contained in a tank 4 feet long, $3\frac{1}{2}$ feet wide, and 1 foot 10 inches deep?

6. How many cubic yards of earth must be removed to dig a cellar 40 feet long, 30 feet wide, and 6 feet 6 inches deep?

7. A circular lake is 100 rods in diameter. How many acres does it cover?

8. A merchant purchased a bill of goods amounting to \$1,872, list price, and was given a discount of $33\frac{1}{3}\%$; with 25 and 5 off additional. Find the actual cost of the goods, and the single discount equal to the three given.

9. How much less will it cost to fence 40 acres of land in the form of a square, than in the form of a rectangular strip, of which the breadth is $\frac{1}{8}$ the length of one side of the square, the price per rod being 75 cents?

10. Find the amount of \$240, exact interest, at $5\frac{1}{2}\%$ per annum, from January 15, 1899, to March 3, 1899.

11. I sold a piece of cloth for \$240, and lost 20%. What selling price would have given me a gain of 20%?

12. A piece of land is 40 rods wide. How long must it be to contain 100 acres? What would be the cost of fencing it at 75 cents per rod?

13. Two boys bought a sled, one paying $\frac{3}{4}$ of a dollar, and the other $\frac{7}{8}$ of a dollar. They sold it for $\frac{3}{16}$ of a dollar more than they paid for it. What did they sell it for, and what was each one's share of the gain?

14. The principal is \$150, the interest is \$14.55, and the rate is 6%. Find the time.

1. Find your exact age today.

2. The consequent is $\frac{2}{3}$, the ratio $\frac{1}{8}$. What is the antecedent?

3. A trade discount of 25% and $16\frac{2}{3}$ is equal to what single discount?

4. At what rate per cent will \$1,651.60 in 3 years 6 months gain \$289.03?

5. A captain observed that when the sun crossed the meridian of his ship, it was 18 minutes 3 seconds past 5 o'clock A. M. by his chronometer, set to Greenwich time. Find the longitude of the ship.

6. A, B, and C formed a partnership. A put in \$3,000 for 1 year, B \$4,500 for 8 months, and C \$1,000 for 6 months. They lost \$4,000. What was each man's share of the loss?

7. For what must one buy 7% stock to receive 8% on his investment?

8. The distance between the opposite corners of a square field is 60 rods. How many acres in the field?

NOTE.—Think of the diagonals of the square forming right-angled triangles.

9. What principal in 3 years 9 months 18 days, at 8%, will give \$114 interest?

10. At what rate will \$450 at interest for 2 years 8 months 10 days amount to \$504.56$\frac{1}{4}$?

11. How many board feet in a plank 18 feet long, 10 inches wide, and $1\frac{1}{2}$ inch thick?

12. Find the amount of \$600 from June 10, 1894, to August 28, 1897, at 5 per cent.

13. A man steps 2 feet 6 inches each step. How many steps will he take in walking 1 mile 1 rod 2 yards 2 feet 6 inches?

14. In a certain firm B has 3 times as much capital as A, and C has $\frac{1}{2}$ as much as the other two. What is each one's share in a loss of \$1,572?

15. A man sold 1 ton 2 hundredweight 20 pounds of wheat at 75 cents a bushel, and spent the money for syrup at 60 cents a gallon. How many gallons did he buy?

From the report of the New York stock market, as given in the daily newspapers, the following table is taken—

	Opening.	Highest.	Lowest.	Closing.
American Sugar	$141\frac{1}{2}$	$147\frac{7}{8}$	$140\frac{3}{4}$	$141\frac{7}{8}$
American Tobacco	$117\frac{1}{2}$	$118\frac{1}{2}$	116	$117\frac{3}{4}$
Chicago, Burlington & Quincy	129	130	$128\frac{7}{8}$	$129\frac{1}{4}$
Union Pacific	44	44	$43\frac{5}{8}$	$43\frac{5}{8}$
Northern Pacific	$52\frac{3}{8}$	$53\frac{7}{8}$	$52\frac{1}{4}$	$52\frac{1}{4}$
Western Union	$87\frac{3}{4}$	$88\frac{5}{8}$	$87\frac{1}{8}$	$87\frac{3}{4}$
Tennessee Coal and Iron	115	$115\frac{1}{4}$	$114\frac{1}{2}$	$114\frac{1}{2}$
Brooklyn Rapid Transit	$87\frac{3}{4}$	$89\frac{1}{8}$	$87\frac{3}{4}$	$88\frac{5}{8}$
Louisville & Nashville	78	$78\frac{1}{4}$	$77\frac{1}{4}$	$77\frac{1}{4}$
Federal steel	$52\frac{1}{2}$	53	$51\frac{3}{4}$	53

Find the gain or loss, brokerage $\frac{1}{8}\%$ each way, buying and selling on—

1. 100 shares American Sugar stock, bought at opening and sold at the highest.

2. 50 shares American Tobacco stock, bought at lowest and sold at closing.

3. 25 shares C., B & Q. stock, bought at the highest and sold at the lowest.

4. 100 shares Union Pacific stock, bought at opening and sold at the closing.

5. 100 shares Northern Pacific stock, bought at opening and sold at closing.

6. 10 shares Western Union stock, bought at the opening and sold at lowest.

7. 100 shares Tennessee Coal and Iron, bought at lowest and sold at highest.

8. 25 shares Brooklyn Rapid Transit, bought at opening and sold at closing.

9. 10 shares Louisville & Nashville, bought at lowest and sold at highest.

What is the interest—

1. Of $ 250.60 for 1 year 9 months, at 6 %?
2. Of $ 956 for 5 years 4 months, at 9 %?
3. Of $1575.20 for 3 years 8 months, at 7 %?
4. Of $5000 for 2 years 3 months, at 5½%?
5. Of $1508.20 for 4 years 2 months, at 10 %?
6. Of $ 75 for 6 years 10 months, at 12½%?
7. Of $ 125 for 5 years 6 months, at 4¾%?
8. Of $4000 for 1 month 6 days, at 9%?
9. Of $8450 for 60 days, at 10%?
10. Of $4500 for 2 years 9 months 20 days, at 5%?
11. Of $ 156.25 for 10 months 18 days, at 8%?
12. Of $ 640 for 3 years 2 months 9 days, at 6½%?
13. Of $ 276.50 for 12 years 11 months 21 days, at 8½%?
14. Of $ 378.42 for 1 year 3 months 5 days, at 7 %?
15. Of $1250 for 7 months 21 days, at 10½%?
16. Of $6500 for 2 months 10 days, at 9½%?

17. If 6 men dig a cellar 22.5 ft. long, 17.8 ft. wide, and 10 ft. 3 in. deep, in 3 days of 10 hours, and 15 minutes each, how many men will it require to dig another, in 12 days of 8.2 hours each, 45 feet long, 35⅜ feet wide, and 12.3 feet deep?

18. What price must be paid for an 8% bond so as to yield 10% on the investment?

19. Which is the better investment, 6% bonds bought at 102, or 4½% bonds at 90? What per cent better?

20. If a farm is bought for ⅝ of its value, and sold for 12% more than its value, what is the gain per cent?

21. When it is 12 o'clock (noon) at Portland, Oregon, what time of day is it at Omaha, 96° W., supposing the longitude of Portland to be 124° W.?

22. A and B rented a pasture for $29.85. A puts in 11 cows for 20 days, and B 31 cows for 25 days. How much rent should each pay?

23. When it is 12:30 A. M. in Paris, what time is it in Washington?

1. How many acres in a rectangular field 30 ch. by 5 ch.?

2. How many acres in a rectangular field 54 ch. by 18 rods?

3. The base of a parallelogram is 542 yards, and the perpendicular height 720 feet. What is the area?

4. The area of a rectangular lot is 14,000 square feet, and its length is 200 feet. How wide is it?

5. At 16 cents a yard, what will it cost to plaster the walls of a room 22 ft. 8 in. long, 18 ft. 9 in. wide, and 11 ft. 6 in. high? There are to be deducted 8 windows, each 6 ft. 4 in. high, and 2 ft. 9 in. wide; 2 doors, each 7 ft. 6 in. high, 3 ft. 2 in. wide, and the base board 1 ft. wide.

6. The people of a school district wish to build a schoolhouse, which shall cost $2,850. The taxable property of the district is valued at $190,000. What will be the tax on a dollar, and what will be a man's tax whose property is valued at $7,500?

7. Find the cost of a piece of land 40 chains 15 links square, at $30 an acre.

8. What is the area of a trapezoid, whose parallel sides are 15 chains and 24.5 chains, and the perpendicular height 30.8 chains?

9. What are the contents of a trapezoid, when the parallel sides are 40 and 64 chains, and the perpendicular distance between them 52 chains?

Find the value of the following expressions:

10.	6^3.	**14.**	4.5^2.	**18.**	6.5^2.	**22.**	$(24\frac{2}{5})^3$.
11.	125^2.	**15.**	6.5^2.	**19.**	$(\frac{3}{8})^4$.	**23.**	$.25^5$.
12.	12^4.	**16.**	$.45^4$.	**20.**	$(2\frac{1}{4})^5$.	**24.**	$(14\frac{1}{4})^4$.
13.	9^5.	**17.**	$(\frac{9}{16})^3$.	**21.**	$(4\frac{3}{8})^5$.	**25.**	$[9^3 \div (\frac{1}{3})^4]^3$.

26. A vessel, in longitude 72° 25′ east, sails 105° 30′ 56″ west, then 46° 50′ east, then 10° 5′ 40″ west, then 39° 11′ 36″ east. In what longitude is she then, and how many days will it take her to sail to longitude 77° west, if she sails 3° 20′ each day?

27. Divide the product of $8\frac{1}{3}$, 12.5, $\frac{1}{8}$, $62\frac{1}{2}$, 140, and 5^3 by the product of $2\frac{1}{3}$, $1\frac{1}{4}$, $6\frac{1}{4}$, $33\frac{1}{3}$, 25^2, and $37\frac{1}{2}$, using cancellation.

28. Find the sum of the exact ages of your classmates.

ALGEBRA.

EQUATIONS.

1. Two boys have \$24, and one has 3 times as much as the other. Find the share of each?

SOLUTION.

Let (1) x = the share of one boy,
Then (2) $3x$ = the share of the other,
(3) $4x$ = \$24, what both had,
(4) x = $\frac{1}{4}$ of \$24, or \$6, the share of one boy.
(5) $3x = 3 \times 6$, or \$18, the share of the other.

2. A father is 3 times as old as his son, and the sum of their ages is 60 years. Find the age of each.

3. A horse and carriage are worth \$300, and the horse is worth twice as much as the carriage. Find the value of each.

4. The sum of 2 numbers is 64, and the greater is 7 times the less. Find the numbers.

5. A farmer had a sheep, a cow, and a horse. The cow was worth twice as much as the sheep, and the horse was worth twice as much as the cow. They all were worth \$140. Find the value of each.

6. Two men, 40 miles apart, travel toward each other at the rate of 4 miles an hour each, what time will they meet?

7. Two men are 28 miles apart. If they travel toward each other, the first at the rate of 3, and the second at the rate of 4 miles an hour, in how many hours will they meet?

8. Two men travel toward each other, at the same rate per hour, from two places, whose distance apart is 48 miles, and they meet in 6 hours. How many miles per hour does each travel?

9. Two men travel toward each other, the first going twice as fast as the second, and they meet in 2 hours. The places are 18 miles apart. How many miles per hour does each travel?

10. James bought a number of lemons, and twice as many oranges, for 40 cents. The lemons cost 2 cents, and the oranges 3 cents apiece. How many were there of each?

AXIOMS.

The solution of a problem in Algebra consists—

1. In reducing the statement to the form of an equation.
2. In reducing the equation to find the value of the unknown quantities.

Operations in Algebra are based upon certain self-evident truths called *Axioms*, of which the following are the most common—

1. If equals are added to equals the sums are equal.
2. If equals are subtracted from equals the remainders are equal.
3. If equals are multiplied by equals the products are equal.
4. If equals are divided by equals the quotients are equal.
5. The whole of a quantity is greater than any of its parts.
6. The whole of a quantity is equal to the sum of all its parts.
7. Quantities respectively equal to the same quantity are equal to each other.
8. Like powers and like roots of equals are equal.

1. A man travels for three days, going twice as far on the second day as on the first, and three times as far on the third day as on the first. He traveled 120 miles in three days. How far did he travel each day?

2. A grocer sold 1 pound of coffee and 2 pounds of tea for $1.08, and the price of a pound of tea was four times that of a pound of coffee. What was the price of each?

3. A grocer sold 1 pound of tea, 2 pounds of coffee, and 3 pounds of sugar, for $1.30 cents. The price of a pound of coffee was twice that of a pound of sugar, and the price of a pound of tea was three times that of a pound of coffee. Find the cost of each.

4. A farmer bought a horse, a cow, and a calf, for $70. The cow cost three times as much as the calf, and the horse twice as much as the cow. What was the cost of each?

5. A horse and buggy cost $240. The buggy cost one half as much as the horse. Find the cost of each.

1. Divide 65 into two parts so that one part is 15 more than the other.

SOLUTION.

Let (1) x = the smaller part,
(2) $x + 15$ = the larger part,
(3) $2x + 15 = 65$,
Subtracting 15, (4) $2x = 50$, (Axiom 2)
(5) $x = 25$, one part,
(6) $25 + 15 = 40$, the other part.

2. Divide the number 15 into two parts, so that one part shall exceed the other by 3.

3. Divide the number 26 into two parts, so that the greater part shall be 5 more than twice the less part.

4. In a school of 40 pupils, there were 8 more girls than boys. How many of each?

5. Find two numbers whose sum is 60 and whose difference is 8.

6. Divide 83 into three parts so that the second is 5 more than the first, and the third is 7 more than the second.

7. A farmer bought a cow and a calf for $13. The cow cost three times as much as the calf, and $1 more. What was the cost of each?

8. William and Thomas gave 50 cents to a poor woman. William gave twice as many as Thomas, and 5 cents more. How many cents did each give?

9. Eliza and Jane bought a doll for 14 cents. Eliza paid twice as much as Jane, and 2 cents more. What did each pay?

10. The sum of two numbers is 23, and the greater is equal to three times the less, and 3 more. What are the numbers?

11. Two numbers added together make 40. The greater is 5 times the less, and 4 more. What are the numbers?

12. A man has two flocks of sheep. The larger contains six times as many as the smaller, and 5 more, and the number in both is 82. How many are there in each?

1. Divide 95 into three parts so that the second is one more than the first, and the third is twice the second.

SOLUTION.

Let (1) x = the first part,
(2) $x+1$ = the second part,
(3) $2x+2$ = the third part,
(4) $4x+3$ = 95,
Subtracting, (5) $4x$ = 92, (Axiom 2)
(6) x = 23, the first part,
(7) $x+1$ = 24, the second part,
(8) $2x+2$ = 48, the third part.

2. If $x+2$ represent a certain number, what will represent twice that number?

3. What is 3 times $2x+1$? 4 times $2x+1$? 5 times $2x+1$?

4. What is 2 times $3x+2$? 3 times $3x+2$? 4 times $3x+2$?

5. What is the sum of x, $x+1$, and $x+2$?

6. What is the sum of x, $x+1$, and $3x+3$?

7. What is the sum of x, $x+3$, and $2x+2$?

8. A father divided 15 cents among his three boys, giving to the second 1 more than to the first, and to the third twice as many as to the second. How many cents did each receive?

9. The sum of three numbers is 34. The second is 1 more than the first, and the third is 3 times the second. What are the numbers?

10. Eliza, Jane, and Sarah, together have 24 cents. Jane has twice as many as Eliza, and 1 more, and Sarah has twice as many as Jane. How many cents has each?

11. A man bought 1 pound of coffee and 2 pounds of tea, for \$1.52, the price of a pound of tea was equal to that of 2 pounds of coffee, and 1 cent more. What was the cost of each?

12. A man worked three days for \$10, the second day he earned \$1 more than the first, and the third day as much as both the first and second. How much did he earn each day?

13. Make an original problem in Algebra.

1. If Willie's age be multiplied by 3, and 30 be added, the sum will be 6 times his age in years. What is his age?

SOLUTION.

Let (1) x = Willie's age.
(2) $3x+30$ years $= 6x$,
Subtracting $3x$, (3) 30 years $= 3x$,
(4) 10 years $= x$, his age.

2. What number is that which is 6 less than 3 times the number itself?

3. James is 12 years younger than John; but John is only 4 times the age of James. What are their ages?

4. What number is that, if to its double 8 be added, the sum will be equal to 4 times the number?

5. What is the value of x, when $5x$ is equal to $3x+6$?

6. What is the value of x, when $5x$ is equal to $2x+15$?

7. What is the value of x, when $8x$ is equal to $3x+15$?

8. What is the value of x, when $10x$ is equal to $4x+24$?

9. What number is that, if to its double 21 be added, the sum will be five times the number?

10. If George's age be multiplied by 4, and 30 added to the product, the sum will be 6 times his age. What is his age?

11. What number added to twice itself and 32 more, will make a sum equal to 7 times the number?

12. What number added to itself and 40 more, will make a sum equal to 10 times the number?

13. A father gave his son 3 times as many cents as he then nad, and his uncle then gave him 40 cents, when he found he had 9 times as many as at first. How many had he at first?

14. A boy found 3 times as many marbles as he first had, and bought 40 more, and then had 8 times as many as he had at first. How many had he at first?

15. Albert is 3 years older than his brother, and twice his age is equal to three times his brother's age. How old is each?

16. What number is 28 less than 5 times itself?

1. Roger is one-fourth as old as his father, and the sum of their ages is 50 years. Find the age of each.

SOLUTION.

Let (1) x = father's age,
(2) $\frac{1}{4}x$ = Roger's age,
Adding, (3) $\frac{5}{4}x$ = 50 years,
Dividing by 5, (4) $\frac{1}{4}x$ = 10 years, Roger's age, (Axiom 4)
Multiplying by 4, (5) x = 40 years, the father's age, (Axiom 3).

Or—

(3) $\frac{5}{4}x$ = 50 years,
Multiplying by 4, (4) $5x$ = 200 years, (Axiom 3)
Dividing by 5, (5) x = 40 years, (Axiom 4)
Dividing by 4, (6) $\frac{1}{4}x$ = 10 years, (Axiom 4).

2. Thomas and Charles have 35 cents, and Charles has half as many more cents as Thomas. How many cents has each?

3. The double of a certain number, increased by one third of itself, is equal to 21. What is the number?

4. William, James, and Robert, together, have 33 cents. James has twice as many as William, and Robert has one third as many as James. How many cents has each?

5. What number is that, which being increased by its half, and its fourth, will be equal to 21?

6. What number is that, which being increased by its half, its fourth, and 4 more, will be equal to 25?

7. A boy, being asked how much money he had, replied, that if one half and one third of his money, and 9 cents more, were added to it, the sum would be 20 cents. How much had he?

8. There are three numbers, whose sum is 44. The second is equal to one third of the first, and the third is equal to the second and twice the first. What are the numbers?

9. There are four towns in the order of the letters A, B, C, and D. The distance from B to C is one fifth of the distance from A to B, and the distance from C to D is equal to twice the distance from A to C The whole distance from A to D is 72 miles. Required the distance from A to B, from B to C, and from C to D.

AN EQUATION is an expression of equality between two quantities. That portion of the equation which precedes the sign (=) is called *the first member*, and that which follows, *the second member*.

THE REDUCTION OF AN EQUATION consists in finding the value of the unknown quantity, and the processes involved depend upon the Axioms given.

TRANSPOSITION is the changing of terms from one member of an equation to the other without destroying the equality.

The object of transposition is to bring all the unknown terms into one member, and all the known into the other.

1. $x+16 = 24$. Find the value of x.

SOLUTION.

(1) $x+16 = 24$

(2) $x \quad = 24-16.$

Subtracting 16 from the first member leaves x; but if 16 is subtracted from the first member, it must also be subtracted from the second. Note that 16 has been transposed from one member to the other member, and the sign is changed.

2. $x-b = a$. Find the value of x.

SOLUTION.

(1) $x - b = a.$

(2) $x = a + b.$

Adding b to the first member gives x; but if b is added to the first member it must also be added to the second. Note that b has been transposed from one member to the other, and the sign is changed.

3. $3x = x+18$. Find the value of x.

SOLUTION.

(1) $3x = x+18$

(2) $3x-x = 18$

(3) $2x = 18$

(4) $x = 9$

Subtracting x from both members, the x is transposed from the second member to the first, and the sign is changed.

Find the value of x—

4. $2x = x+8.$

5. $3x = 32-x.$

6. $x+20 = 8x-8.$

7. $5x-5 = 3x+3.$

8. $5x+6 = x+26.$

9. $8x-8 = 2+6x.$

10. $57 - 3x = 6x+22-2x.$

11. $20 - 2x = x+2.$

DEFINITIONS.

UNKNOWN QUANTITIES are represented by the last letters of the alphabet, x, y, z, etc.; other quantities are sometimes represented by the first letters, a, b, c, etc.

NUMERICAL QUANTITIES are those expressed by figures.

LITERAL QUANTITIES are those expressed by letters, as a, x, y, etc.

MIXED QUANTITIES are those expressed by both figures and letters, as $3a$, $4x$, etc.

The words *plus* and *minus*, *positive* and *negative*, and the signs + and —, have a merely *relative* signification; thus, the navigator and the surveyor always represent their northward and eastward progress by the sign +, and their southward and westward progress by the sign —. So also is represented gain and loss, above and below zero, credit and debt.

If a man's *property* is considered *positive*, his *gains* should also be considered *positive*, while his *debts* and his *losses* should be considered *negative*. Suppose a man has a farm worth \$5,000, and other property worth \$3,000, and that he owed \$4,000; the net value his estate is \$5,000+\$3,000—\$4,000 = \$4,000. Again, suppose his farm is worth \$5,000, and his other property \$3,000, and he owes \$12,000; his net estate is worth \$5,000+\$3,000—\$12,000 = —\$4,000, *i. e.*, he owes \$4,000 more than he can pay. From this last illustration it is seen that the sign — may be placed before the quantity standing alone without indicating what it is to be subtracted from.

THE TERMS of an algebraic expression are the quantities which are separated from each other by the signs + or —; thus, in the equation $4a-b = 3x+c$, the first member consists of the two terms $4a$ and $-b$, and the second of $3x$ and c.

A COEFFICIENT is a number or letter prefixed to a quantity to show how many times that quantity is to be taken; thus, in the expression $4x$, which equals $x+x+x+x$, the 4 is the coefficient of x.

ADDITION OF MONOMIALS.

ADDITION is the process of combining two or more algebraic quantities into one quantity.

In adding similar terms, add the coefficients of like quantities.

SIMILAR TERMS are those which have *the same powers of the same letters*, as x and $3x$, or $5ax^3$ and $-2ax^3$.

1. One boy had 4 marbles, a second had 5 marbles, and a third had 6 marbles. How many had they together?

SOLUTION.

4 marbles	It is evident that the sum is 15 marbles.	$4x$
5 marbles	If for the marbles an unknown quantity, x,	$5x$
6 marbles	is substituted, the addition is performed in	$6x$
15 marbles	the same manner by adding the coefficients.	$15x$

Find the sum of

2.	**3.**	**4.**	**5.**	**6.**	**7.**
$5a$	$3a^2$	$4x$	$6y$	$-3x^3$	$-5by$
$8a$	$4a^2$	$7x$	$10y$	$-2x^3$	$-2by$
$4a$	$7a^2$	$5x$	y	$-7x^3$	$-by$
$2a$	$3a^2$	$3x$	$2y$	$-4x^3$	$-by$
$19a$	$6a^2$	$19x$	$8y$	$-6x^3$	$-4by$

8. Find the sum of $5a$, $-2a$, $6a$, $-4a$, and $3a$.

SOLUTION.

$+5a$	The sum of the positive quantities $= +14a$.
$-2a$	The sum of the negative quantities $= -6a$.
$+6a$	The sum $= 8a$.
$-4a$	Notice that the sum takes the sign of the
$+3a$	larger coefficient.
$8a$	

9.	**10.**	**11.**	**12.**	**13.**	**14.**
$13x$	$14y^2$	$13ab$	$24y$	$25(a-6)$	$-8(x+y)$
x	$-2y^2$	$16ab$	$-13y$	$-50(a-6)$	$-4(x+y)$
$-5x$	$7y^2$	$-4ab$	$20y$	$30(a-6)$	$17(x+y)$
$17x$	$-3y^2$	$-8ab$	$-y$	$-47(a-6)$	$-3(x+y)$
$-2x$	$24y^2$	$14ab$	$20y$	$18(a-6)$	$(x+y)$

ADDITION OF POLYNOMIALS.

If the terms are not similar, indicate the sum by the proper sign. To add a and x, it is evident that they are not similar terms and cannot be combined by adding their coefficients. Hence, this sum is indicated: $a+x$. Likewise, the sum of a, $-b$, and c is indicated: $a-b+c$. This gives rise to terms formed by connecting other terms.

A MONOMIAL is a single term; as $3ax$.

A POLYNOMIAL is a number of terms connected by the signs plus or minus; as $x+y$, $3a+4x-7y$.

A BINOMIAL is a polynomial of two terms; as $3x+3y$, or $x-y$.

1.

$$\begin{array}{r} 4x-7a+3y-4b+3z \\ 6a-\ y+4b-3z \\ 4a-2y+8b-\ z \\ \hline -3a \qquad -8b \qquad -10c \end{array}$$

2.

$$\begin{array}{r} 6x+2b-3y+2\sqrt{a} \\ -\ b+4y-4\sqrt{a} \\ -6y+5\sqrt{a} \\ \hline \sqrt{a} \end{array}$$

3. Add $7x-3y-5b+4c$, $3ax+4x+5b-5c$, and $3c-3ax+7y+c$.

4. Add $5a-2+6x+4ax-3ab$, $6ab-4a+2-4ax+6$, and $6-4ab+4x+14y+ax$.

5. $16xy+8xz-5mn+2n$, $4mn-6xy+3n-7mn$, $-6xz+14n-13xy+8$, and $12mn-11n+4-6$.

6. Add $4am+9nx-5b+c$, $-19v+14b-16c+y$, and $18nx-4am+15v+4y$.

7. Add $17x^2+19x^3-14x^4+16x^2$, $13x^3-5x^4+6x^2-10x^3$, and $14x^4+17x^3-3x^3+15x^2$.

SUBTRACTION.

Note the result of the following addition—

4	4	4	4	4	4	4	4	4	4
4	3	2	1	0	—1	—2	—3	—4	—5
8	7	6	5	4	3	2	1	0	—1

As the lower number decreases, the sum decreases. If zero is added 4, the 4 is unchanged. If 1 less than zero (—1) is added, the 4 is decreased to 3; if 2 less than zero (—2) is added, the 4 is decreased to 2, etc.

If 4 less than zero (—4) is added, the sum becomes 0; that is, if a +4 and a —4 are added, they cancel each other, and *nothing* is left.

If the lower number is decreased again by 1, and becomes —5, the sum is also decreased by 1, and becomes 1 less than nothing, or —1; that is, if the *negative* quantity is numerically greater than the *positive*, the sum will be a *negative* quantity.

Note the following subtraction—

3	3	3	3	3	3	3
3	2	1	0	—1	—2	—3
0	1	2	3	4	5	6

It is seen that 3 from 3 leaves nothing. If the subtrahend 3 is decreased by 1, and becomes 2, the difference becomes 1 greater than nothing; if the subtrahend becomes 0, the difference is increased to 3; if the subtrahend is decreased to —3, the difference becomes 6. That is, subtracting a *negative* quantity gives the directly opposite result from subtracting a *positive* quantity. It is seen from the above operation that the difference of 6 is obtained by changing the — to a + and *adding.*

If I have $1,000 and owe $400, it is the same to me if a friend pays the debt of $400, or gives me $400. In either case I shall be worth $1,000. In the former case he *subtracts* a *negative* quantity; in the latter he *adds* a *positive.* This leads to the general principle, that the subtraction of any quantity gives the same result as the addition of that quantity with *the opposite sign.*

1. One boy earned 15 cents on a certain day, and another boy lost 10 cents. What was the difference between their earnings of the day?

SOLUTION.

+15 cents
−10 cents
25 cents.

It is evident that the resulting 25 cents is the difference. That is, 15 cents −(−10 cents)= 25 cents.

2.	**3.**	**4.**	**5.**	**6.**	**7.**
$10x$	$-4xy$	$-4ab$	$+6c$	$-25xy$	$27b$
$6x$	$27xy$	$-13ab$	$-18c$	$9xy$	$-38b$

8.	**9.**	**10.**	**11.**	**12.**	**13.**
$6x$	$27axy$	$-13ab$	$-18c$	$9xy$	$-38b$
$10x$	$-4axy$	$-4ab$	$+6c$	$-25xy$	$27b$

14. From 14, take $ab-5$.
15. From $a+b$, take a.
16. From a, take $a+b$.
17. From x, take $x-5$.
18. From $3ax$, take $2ax+7$.
19. From $x+y$, take $x-y$.
20. From $x-y$, take $x+y$.
21. From $x-y$, take $y-x$.
22. From a, take $-a$.
23. From $8a$, take $-3a$.
24. From a, take $-4a$.
25. From $5b$, take $11b$
26. From a, take $-b$.
27. From $3a$, take $-2b$.
28. From $-9a$, take $3a$
29. From $-7a$, take $-7a$.
30. From $-6a$, take $-5a$.
31. From $-3a$, take $-5b$.
32. From -13, take 3.
33. From -9, take -16.
34. From 12, take -8.
35. From -14, take -5.
36. From $84x-75y+28c$, take $18x+75y-7c$.
37. From $x^2-y^3+x^4-10x^3$, take $x^2+4x^3-x^4-4y^3$.

38.	**39.**
$16x-24y+3z$	$7a+8b-10c$
$13x+23y+z$	$-25b+6c-8$

USE OF PARENTHESIS.

By the use of the *signs of aggregation* two or more algebraic quantities may be combined and considered as a single quantity. Thus, $3a+3b$ may be expressed by combining a and b, and giving the coefficient to the resulting quantity, as $3(a+b)$, which is read 3 times the *quantity* $a+b$.

In the solution of many problems it is convenient sometimes to unite quantities by the parenthesis or other sign of aggregation, and sometimes to remove such sign and consider each single term separately.

Thus, $a+b+c=a+(b+c)$; from which it is apparent that where the signs are *plus* the parenthesis makes no change in the result. It simply changes the order in which the quantities are added.

In the case of *minus* quantities it is different.

Thus, $a-b+c$ indicates that b is subtracted from a, and c is added to the difference. But if b and c are combined by parenthesis, the expression would be $a-(b+c)$, which indicates that both b and c, or the *sum* of b and c is subtracted from a. It is apparent that if the parenthesis is removed without changing the value of the expression, the signs must be changed to indicate the subtraction of both quantities, as $a-(b+c)=a-b-c$.

In this manner it is shown that if a parenthesis is removed after a negative sign, *the signs within the parenthesis are changed*.

Remove parenthesis—

1. $a+(b-c-d) =$
2. $a+b-(c-d) =$
3. $a+b-(c+d)+c =$
4. $a-(b+c+d)+b =$
5. $a-(b+c-d)-a =$

Enclose all but first term—

6. $a-b+c =$
7. $b+c-d =$
8. $x^2-2xy+z =$
9. $ax+bc-cd+h =$
10. $m-n-z-s =$

Enclose the second and fifth terms in brackets, and the third and fourth terms in parenthesis, without changing the value of the expression—

11. $a-b-c+d-e.$
12. $a-b-c-d-e.$
13. $b-4-c+y-z.$
14. $ad-ad+b-2b-c.$

MULTIPLICATION.

Multiplication of algebraic terms is governed by the same general principles as is multiplication in arithmetic.

In algebra, the sign of the multiplier shows whether the repetitions are to be added or subtracted.

In multiplication the sign of the product is determined by the general principle that *like signs give* +, *unlike signs give* —.

Hence, the product of an *even* number of negative factors is positive; of an *odd* number, negative.

In finding the product of *monomials* multiply the numerical factors of the terms and annex the literal factors.

Thus, $(+4x)\times(+3y) = +12xy.$

$-3a \times -4b = +12ab.$

When a literal factor is repeated in a product, the number of times the factor occurs is indicated by a small figure written a little above and to the right of the letter.

Thus the product of $4a$ and $5a$ is written $+20a^2$.

The product of aa is written a^2; of aaa, a^3; of $aaaa$, a^4; and so on.

$(a+b)$ multiplied by $(a+b)$, indicated by $(a+b)\ (a+b)$, is also written $(a+b)^2$.

$(a+b)\ (a+b)\ (a+b)$, is also written $(a+b)^3$.

The products, a^2, a^3, a^4, are called the second, third, and fourth *powers* of a.

The number which indicates the power is called the *index*, or *exponent*. When a letter has no index, 1 is to be understood; thus, $a = a^1$.

The product of a^2 and a^3 is $a^{2+3} = a^5$.

1. What is $2a - a^2$ equal to, when a is 1? When a is 2?
2. What is $a^2 - 2a$ equal to, when a is 5? When a is 6?
3. What is $a^3 - 3a$ equal to, when a is 4? When a is 5?
4. What is $a^4 - 4a$ equal to, when a is 3? When a is 4?
5. What does $x^2+2\times9+7^2$ equal when x is ady is 8?
6. What does $x^2-2\times7+7^2$ equal when x is ady is 5?
7. What does $(a^2-b^2)\ (a-b^2)$ equal when a is 6 and b is 3?
8. What does $(a+b^2)\ (a^2+b^2)$ equal when a is 6 and b is 5?

MULTIPLICATION OF MONOMIALS.

1. Multiply $4ab$ by $5xy$.
2. Multiply $7ax$ by $4cd$.
3. Multiply $6by$ by $3ax$.
4. Multiply $3a^2b$ by $4ab$.
5. Multiply $2xy^2$ by $3x^2y$.
6. Multiply $4ab^2x$ by $5ax^2y$.
7. Multiply $-3b$ by $+4b$.
8. Multiply $-2mn$ by $-5mn$.
9. Multiply $-5x^3$ by $-3x^4$.
10. Multiply $-5xy$ by $-3x$.
11. Multiply $-3b^2$ by $+4b^3$.
12. Multiply $-2m^{10}$ by $5m^6$.
13. Multiply $6x-5x$ by 3.

SOLUTION.

$$\begin{array}{l} 6x-5x \\ 6x-5x \\ \underline{6x-5x} \\ 18x-15x \end{array} \qquad \begin{array}{r} 6x-5x=x \\ 3\quad 3 \\ \hline 18x-15x=3x \end{array}$$

The product of 3 times $6x=18x$.
The product of 3 times $5x=15x$.
The product of 3 times $x=3x$.

To find the product of $(a+b)$ and c, we must add the product of a and c, and the product of b and c.

Thus, $$(a+b)\,c = ac+bc.$$

This result is true, no matter what values are given to a and b.

Hence the product of a polynomial and a monomial is obtained by multiplying each term of the polynomial by the monomial.

Multiply—

14. $a+b$ by x.
15. $3c+5d$ by m.
16. $2a+3b$ by $-2x$.
17. $c+d$ by m.
18. $2a+3b$ by $4x$.
19. $3c+5d$ by $-3m$.
20. $2a+3b$ by x.
21. $3c+5d$ by $10m$.
22. $3a-4b$ by $2c$.
23. $2c-4d$ by 4.
24. x^2+x by $2x$.
25. x^2+2x+1 by x.
26. x^2+xy+y^2 by x^2.
27. x^2+xy+y^2 by y^2.
28. $2x^2-3x$ by $3x$.
29. x^2-2x-2 by $-xy$.
30. x^2-xy+y^2 by xy.
31. $2a+3b-4c$ by $2abc$.

A polynomial in parenthesis is considered as a monomial.

Thus, 4 $(a+b)$ times 3 $(a+b)$ = 12 $(a+b)^2$.

32. Multiply $-4\,(a^2-b^2)$ by $-3\,(a^2-b^2)$.
33. Multiply $(a-x)^3$ by $(a-x)^4$.
34. Multiply $4\,(a+b)^5$ by $2\,(a+b)$.

MULTIPLICATION OF POLYNOMIALS.

1. Find the product of $(a+b)$ and $(a+b)$.

SOLUTION.

$$\begin{aligned} a \text{ times } (a+b) &= a^2+ ab \\ b \text{ times } (a+b) &= \quad ab+b^2. \\ \hline (a+b) \text{ times } (a+b) &= a^2+2ab+b^2. \end{aligned}$$

2. $(2x+3y)$ times $(3x+2y)$.

SOLUTION.

$$\begin{aligned} 2x \text{ times } (3x+2y) &= 6x^2+ 4xy \\ 3y \text{ times } (3x+2y) &= \quad + 9xy+6y^2. \\ \hline (2x+3y) \text{ times } (3x+2y) &= 6x^2+13xy+6y^2. \end{aligned}$$

The quantities are usually written in the following form:

$$\begin{array}{rl} & 2x + 3y \\ & 3x + 2y \\ \hline \text{Multiplying by } 3x, & 6x^2+ 9xy \\ \text{Multiplying by } 2y, & \quad 4xy+6y^2 \\ \hline & 6x^2+13xy+6y^2 \end{array}$$

From the above solutions it is evident that the product of two polynomials can be found by taking the sum of all the products found by multiplying each term of an expression by each term of the other.

Find the product of—

3. $a+6$ and $a+15$.
4. $x+4$ and $x+15$.
5. $x+2$ and $x+30$.
6. $x-12$ and $x-5$.
7. $x-4$ and $x-15$.
8. $x-2$ and $x-30$.
9. $x-12$ and $x+5$.
10. $x-4$ and $x+15$.
11. $x-6$ and $x+10$.
12. $x-3$ and $x+20$.
13. $x-2$ and $x+30$.
14. $x-1$ and $x+60$.
15. $7x-4$ and $3x+5$.
16. $x+3$ and $x+20$.
17. $x+1$ and $x+60$.
18. $x-6$ and $x-10$.
19. $x-3$ and $x-20$.
20. $x-1$ and $x-60$.
21. $x+12$ and $x-5$.
22. $x+4$ and $x-15$.
23. $x+6$ and $x-10$.
24. $x+3$ and $x-20$.
25. $x+2$ and $x-30$.
26. $x+1$ and $x-60$.

Multiply—

1. $2x+3y$ by $3a+2b$.
2. $2a+3b$ by $3c+d$.
3. $m+n$ by $x+z$.
4. $4a+3b$ by $2a+b$.
5. $4x+5y$ by $2a+3x$.
6. $2x+3y$ and $x+2y$.
7. $2a+3b$ and $a+2b$.
8. $2m+3n$ and $m+2n$.
9. $2a-b$ and $a+2b$.
10. $2m-n$ and $m+2n$.
11. $3x+24$ and $3x+1$.
12. $3x+12$ and $3x+2$.
13. $x^2+2xy+y^2$ by $x+y$.
14. y^2-y+1 by $y+1$.
15. x^2+y^2 by x^2-y^2.
16. a^2-3a+8 by $a+3$.
17. $2x^2-3xy+y^2$ by x^2-5xy.
18. $3x+y$ and $2x+5y$.
19. $3p+q$ and $2p+5q$.
20. $3x-2y$ and $2x-3y$.
21. $3m-2n$ and $3m-3n$.
22. $3x-24$ and $3x+1$.
23. $3x+12$ and $3x-2$.
24. $3x-8$ and $3x+3$.
25. $3x+2y$ by $2x+3y$.
26. a^2+b^2 by $a+b$.
27. $3a^2+2b^2$ by $2a^2+3b^2$.
28. a^2+ab+b^2 by $a+b$.
29. c^3+d^3 by $c+d$.
30. $3x-8$ and $3x-3$.
31. $x+y$ and $x-y$.
32. $m+n$ and $m-n$.
33. $2x+y$ and $2x-y$.
34. $ab+c$ and $ab-c$.
35. a^2+b^2 and a^2-b^2.
36. $3a+b$ and $2a+5b$.
37. $3a+5b$ by $3a-5b$.
38. $2a^2-4ax+2x^2$ by $3a-3x$.
39. $5x^3+3y^3$ by $5x^3-3y^3$.
40. $3a^2+3ax+3x^2$ by $2a^2-2x$.
41. $3a^2+5ax-2x^2$ by $2a-x$.
42. $3x-10$ and $3x-2$.
43. $a+b$ and $a-b$.
44. $p+q$ and $p-q$.
45. $2x+3y$ and $2x-3y$.
46. $pq+bm$ and $pq-bm$.
47. a^2+bc and a^2-bc.
48. $3x^3+8$ and $3x^2+5$.
49. $(a+b)$, $(b+c)$ and $(c+a)$.
50. $(x+2)$, $(x+3)$ and $(x+4)$.
51. $(x-2)$, $(x-3)$ and $(x-4)$.
52. $(x-4)(x-5)(x+4)(x+5)$.
53. $(a+c)(a-c)(a+c)(a-c)$.
54. $(x+2y)$, $(x+3y)$ and $(x+4y)$.
55. $(x-2y)$, $(x-3y)$ and $(x-4y)$.
56. $(a^2+b^2+c^2-ab-ac-bc)(a+b+c)$.
57. $(n^2+n+1)(n^2+n+1)(n-1)(n-1)$.

DIVISION OF MONOMIALS.

In division of algebraic expressions the sign of the quotient must be + when the divisor and the dividend have like signs; —, when the divisor and dividend have unlike signs; *i. e.*, in division as in multiplication we have the principle that ***like signs give +, unlike signs give —.***

This may be illustrated by the following simple problems—

$3 \times 4 = +12$; therefore, $+12 \div 3 = 4$.
$3 \times (-4) = -12$; therefore, $-12 \div 3 = -4$.
$(-3) \times (-4) = +12$; therefore, $+12 \div (-3) = -4$.
$(-3) \times 4 = -12$; therefore, $-12 \div (-3) = 4$.

Also—

$a \times b = ab$; therefore, $ab \div a = b$.
$a \times (-b) = -ab$; therefore, $-ab \div a = -b$.
$-a \times (-b) = ab$; therefore, $ab \div (-a) = -b$.
$-a \times b = -ab$; therefore, $-ab \div (-a) = b$.

The quotient of a polynomial by a monomial can be found by dividing each term of the dividend by the divisor.

To divide $2ab - 6a^2$ by $2a$,
$+2ab$ divided by $2a$ gives b,
and $-6a^2$ divided by $2a$ gives $-3a$;
$\therefore$ the quotient is $b-3a$.

The quantities are usually written in the following form—

$$2a)\overline{2ab-6a^2}$$
$$\overline{b-3a.}$$

Divide—

1. ab by b.
2. $3ab$ by a.
3. $4ab$ by b.
4. $4a^3$ by $2a^3$.
5. $16x^2$ by $8x$.
6. $15ax^2y^3$ by $3ay$.
7. $6x+12y$ by 3.
8. $15x-20b$ by 5.
9. $21a+35b$ by -7.
10. $6ax+9ay$ by $3a$.
11. $ab+ac$ by a.
12. $4x^2y^3z^4$ by $4xyz^2$
13. $-2x^2yz^3$ by xz^3.
14. $18a^2b^3c^4$ by $-9ab^2c$.
15. $-a^3b^2c^2d$ by $abcd$.
16. $99a^4b^4x^5$ by $11a^3b^2x^4$.

DIVISION OF POLYNOMIALS.

1. Divide $x^3+3x^2y+3xy^2+y^3$ by $x+y$.

SOLUTION.

Dividend.	Divisor.
$x^3+3x^2y+3xy^2+y^3$	$(x+y$
$x^2+\ x^2y$	$x+2xy+y^2$
$2x^2y+3xy^2$	
$2x^2y+2xy^2$	
xy^2+y^3	
xy^2+y^3	

See that the terms of the dividend and quotient are arranged in the same order.

The quotient is written below the divisor to keep them near each other for convenience in multiplying.

The first term of the dividend contains the first term of the divisor how often?

Multiply the divisor by the quotient and subtract the product from the dividend, as in division of numbers, and proceed in the same manner with each remaining dividend, until there is no remainder.

Divide—

2. $x^2+8x+15$ by $x+3$.
3. $x^2-8x+15$ by $x-3$.
4. $x^2+2x-15$ by $x+5$.
5. $x^2-2x-15$ by $x-5$.
6. $x^2+9x+14$ by $x+2$.
7. $x^2-7x+12$ by $x-3$.
8. $x^2-2x-63$ by $x+7$.
9. $x^2-9x-22$ by $x+2$.
10. $x^2+3x+\ 2$ by $x+1$.
11. $x^2+5x+\ 6$ by $x+2$.
12. $x^2+4x+\ 4$ by $x+2$.
13. $a^2+5a+\ 4$ by $x+4$.
14. $x^2+7x+\ 6$ by $x+1$.
15. $x^2-5x-14$ by $x+2$.
16. $x^2-7x-18$ by $x-9$.
17. $c^2-3c-18$ by $c+3$.
18. $x^2+3xy+2y^2$ by $x+2y$.
19. $x^2-5xy-14y^2$ by $x-7y$.
20. $a^3-9a^2+27a-27$ by $a-3$.
21. $4a^4-5a^2x^2+x^4$ by $2a^2-3ax+x^2$.
22. $a^5-5a^4x+10a^3x^2-10a^2x^3+5ax^4-x^5$ by $a^2-2ax+x^2$.
23. $4a^6-25a^2x^4+20ax^5-4x^6$ by $2a^3-5ax^2+2x^3$.

To find the square of a binomial:

$$\begin{array}{r} a+b \\ a+b \\ \hline a^2+ab \\ +ab+b^2 \\ \hline a^2+2ab+b^2 \end{array} \qquad \begin{array}{r} a-b \\ a-b \\ \hline a^2-ab \\ -ab+b^2 \\ \hline a^2-2ab+b^2 \end{array}$$

From the first example it is seen that the square of the sum of two quantities equals the square of the first, plus twice their product, plus the square of the second.

From the second example it is seen that the square of the difference of two quantities equals the square of the first, minus twice their product, plus the square of the second.

Find without ordinary multiplication, the square of—

1. $x+y$.
2. $m+n$.
3. $y+3z$.
4. $x-y$.
5. $m-n$.
6. $2y-3z$.
7. $3m-4n$.
8. $ab+c$.
9. $xb+2c$.
10. $xy+5$.
11. $(2x+3y)$.
12. $(ab+cd)$.
13. (x^2+xy).
14. $m-4a$.
15. $x+\frac{2}{3}$.
16. $x-\frac{3}{5}$.
17. $x+\frac{3}{2}$.
18. $x+\frac{1}{2}$.
19. $y+z$.
20. $2x+y$.
21. $m+4n$.
22. $y-z$.
23. $2x-y$.
24. $(5-4)^2$.
25. $(2a-b)^2$.
26. $(3x-2y)$.
27. $x-\frac{y}{2}$.
28. $2+\frac{x}{2}$.
29. $20+3$.
30. $30+4$.
31. $50+2$.
32. $(100+1)$.
33. $10\frac{1}{2}$.
34. $9\frac{1}{2}$.
35. $99\frac{1}{2}$.
36. $100\frac{1}{2}$.
37. (x^2-y^2).
38. $(ax-x^2)$.
39. $(5a^2-b^2)$.

40. What must be added to x^2+y^2 to make the square of $x+y$?

41. What must be added to x^2+y^2 to make the square of $x-y$?

42. What must be added to x^2+2xy to make the square of $x+y$?

43. What must be added to x^2+4xy to make the square of $x+2y$?

Form a square by adding a term to each of the following, and state the expression of which each is then the square:

1. m^2+2mn.

SOLUTION.

Adding n^2, the expression becomes $m^2+2mn+n^2 = (m+n)^2$.

2. m^2+6m.
3. m^2-2mn.
4. m^2+2mn.
5. m^2-6m.
6. x^2-16x.
7. x^2+3x.
8. x^2y^2+4xy.
9. m^2+n^2.
10. p^2+2pq.
11. p^2-8p.
12. y^2+2y.
13. y^2-2y.
14. y^2-5y.
15. $4a^2b^2+8ab$.
16. p^2+q^2.
17. $4x^2+4xy$.
18. $4x^2+12x$.
19. $4x^4-4xy$.
20. $4x^2-12x$.
21. z^2-4x.
22. $4z^2+5z$.
23. p^2q^2-5pq.

Express each of the following as the square of a binomial—

24. $x^2+2xy+y^2$.
25. $x^2+4xy+4y^2$.
26. $4x^2+4x+1$.
27. $a^2+2abc+b^2c^2$.
28. $\frac{x^2}{4}+x+1$.
29. $a^2+22a+121$.
30. $m^2-12m+36$.
31. $x^2-16xy+64y^2$.
32. $x^2-10xy+25y^2$.
33. $4x^2-12xy+9y^2$.
34. $81x^2+18xy+y^2$.
35. $25x^2-30xy+9y^2$.
36. $x^2+8x+16$.
37. $4x^2-12x+9$.
38. $x^2y^2-8xy+16$.
39. $x^2+ax+\frac{a^2}{4}$.
40. $\frac{x^2}{y^2}+2+\frac{y^2}{x^2}$.
41. $\frac{4m^2}{9}+\frac{4m}{3}+1$.
42. $\frac{9x^2}{16}-2+\frac{16}{9x^2}$.
43. $(a+b)^2+2\,(a+b)\,c+c^2$.
44. $121-22a+a^2$.
45. $36+12x+x^2$.
46. $64x^2-16xy+y^2$.
47. $25y^2-10xy+x^2$.
48. $9x^2+12xy-4x^2$.
49. $x^2+18x+81$.
50. $9y^2+30xy+25x^2$.
51. $1+4x+4x^2$.
52. $9x^2+12x+4$.
53. $16x^2y^2+8xy+1$.
54. $\frac{x^2}{4}-ax+a^2$.

To find the product of the sum and difference of two quantities.

$a + b$ = sum of a and b

$a - b$ = difference of a and b

$a^2 + ab$

$-ab - b^2$

$a^2 \quad -b^2$ = difference of the squares of a and b.

From this it is evident that the product of the sum and difference of two quantities equals the difference of their squares.

Find the product of—

1. $x+y$ and $x-y$.
2. $b+c$ and $b-c$.
3. $m+n$ and $m-n$.
4. $b+2c$ and $b-2c$.
5. $x+4$ and $x-4$.
6. $x+7$ and $x-7$.
7. $x+1$ and $x-1$.
8. $1+2x$ and $1-2x$.
9. $2+2x$ and $2-2x$.
10. a^2+b^2 and a^2-b^2.
11. $a+\frac{1}{2}$ and $a-\frac{1}{2}$.
12. $x+27$ and $x-27$.
13. $ab+\frac{1}{2}$ and $ab-\frac{1}{2}$.
14. $3-2x$ and $3+2x$.
15. $4a-5b$ and $4a+5b$.

Conversely, there can always be found the two factors which give a product expressed as the difference of the squares of two quantities.

Thus, $x^2-y^2 = (x+y)\,(x-y)$.

Find the factors of—

16. c^2-d^2.
17. m^2-n^2.
18. q^2-r^2.
19. $4-x$.
20. $9-4y^2$.
21. $16-a^4$.
22. 27^4-23^4.
23. $121-x^2$.
24. $400-a^2$.
25. $36-25a^2$.
26. $49-100x^2$.
27. $a^2x^2-y^2$.
28. $x^2-y^2z^2$.
29. $x^2y^2-z^2w^2$.
30. $4a^2b^2-x^2y^2$.
31. $25-16p^2q^2$.
32. a^4-b^4.
33. $625x^6-y^4$.
34. $9x^2-16y^2$.
35. $a^2b^2-4c^2d^2$.
36. a^2-64b^6.
37. $9-x^6y^6$.
38. $4m^2-n^2$.
39. $9p^2-4q^2$.
40. $16x^4-9y^4$.
41. $1-\frac{x^2}{4}$.
42. $\frac{1}{25}-y^6z^6$.
43. $64x^2-25$.
44. $49x^4-36$.
45. $36x^4-49$.
46. $64x^2-1$.

To find the product of two binomials which have a common term.

Note the following products:

$$\begin{array}{r} x+5 \\ x+7 \\ \hline x^2+5x \\ \quad +7x+35 \\ \hline x^2+12x+35. \end{array} \qquad \begin{array}{r} x-5 \\ x-7 \\ \hline x^2-5x \\ \quad -7x+35 \\ \hline x^2-12x+35. \end{array}$$

$$\begin{array}{r} x-5 \\ x+7 \\ \hline x^2-5x \\ \quad +7x-35 \\ \hline x^2+2x-35. \end{array} \qquad \begin{array}{r} x+5 \\ x-7 \\ \hline x^2+5x \\ \quad -7x-35 \\ \hline x^2-2x-35. \end{array}$$

It is seen that the product consists of three terms:

(1) *The square of the common term.*

(2) *The product of the common term and the sum of the unlike terms.*

(3) *The product of the unlike terms.*

Find, without ordinary multiplication, the product of—

1. $x+2$ and $x+3$.
2. $x+7$ and $x+11$.
3. $x-3$ and $x-5$.
4. $z+2$ and $z-3$.
5. $a+7$ and $a-11$.
6. $a-3$ and $a+5$.
7. $x+3$ and $x+5$.
8. $x-2$ and $x-3$.
9. $x-7$ and $x-11$.
10. $z+3$ and $z-5$.
11. $a-2$ and $a+3$.
12. $a-7$ and $a+11$.

Find the factors of—

13. x^2+5x+6.
14. $a^2+7a+12$.
15. x^2-5x+6.
16. $x^2-9x+20$.
17. x^2+x-6.
18. x^2-x-6.
19. $x^2+4x-12$.
20. $x^2-2x-63$.
21. $x^2-4x-21$.
22. x^2+x-2.
23. $x^2-13x+40$.
24. x^2-7x-8.
25. $x^2+7x-18$.
26. x^2-x-30.
27. $x^2+10xy+21y^2$.
28. $x^2-10x+24$.
29. $x^2-10x+16$.
30. $x^2-10x+24$.

To clear an equation of fractions.

1. Find the value of x in the equation $\frac{x}{3}-2=\frac{x}{6}+1$.

SOLUTION.

$\frac{x}{3}-2=\frac{x}{6}+1$

$2x-12=x+6$

$x=18$

Multiplying by 6 the least common multiple of 6 and 3, gives $2x-12=x+6$, an equation without a fractional term. It is sometimes better to transpose before clearing of fractions.

The above equation, by transposing, becomes $\frac{x}{3}-\frac{x}{6}=3$.

An equation may be cleared of fractions by multiplying it first by one denominator, and the resulting equation by another, and so on, till all the denominators disappear; or by multiplying by the least common multiple of the denominators.

Algebraic fractions are governed by the same principles that govern numerical fractions, and are computed or reduced by the same methods.

2. Given $\frac{x}{4}-\frac{x}{5}+25=33-\frac{x-6}{2}$ to find x.

SOLUTION.

Transposing 25,	(1)	$\frac{x}{4}-\frac{x}{5}=8-\frac{x-6}{2}$
Multiplying by 20,	(2)	$5x-4x=160-10x+60$
Transposing and uniting,	(3)	$11x=220$
Dividing by 11,	(4)	$x=20$

Find the value of x—

3. $\frac{2}{3}x=8$.

4. $\frac{3}{4}x=10$.

5. $\frac{2}{3}x+10=\frac{1}{3}x+2\frac{1}{2}$.

6. $\frac{2x}{3}=12$.

7. $\frac{2x}{4}=15$.

8. $\frac{x}{5}=6$.

9. $\frac{3x}{4}+\frac{2x}{5}=10\frac{1}{2}+x$.

10. $\frac{4x}{5}+10=\frac{7x}{10}+13$.

11. $x-14=12x-234$.

12. $\frac{x+8}{4}=\frac{x+2}{6}$.

13. $\frac{x}{4}+\frac{x}{6}+\frac{5x}{12}=\frac{3}{2}$.

When a factor is common to all the terms within a parenthesis, or other sign of aggregation, that factor can be removed from each term and placed outside the parenthesis.

Thus, $(ax+bx) = (a+b)\,x$.

Place in parenthesis the terms which contain like powers of x, and then remove that power of x outside of the parenthesis.

1. $4mx^2+5nx^2$.
2. $2ax+5dx+cx$.
3. $ax+bx+cx^2+dx^2$.
4. $2x+3x^2+5cx+4mx^2$.
5. $mx^3+4mnx+nx^3+pqx$.
6. $2x+cx^3+dx+ex^2+fx^3+4x^2$.
7. $a^3bx+a^2bx^2+a^2x^3+a^2b^2x+ab^2x^2$.

By actual multiplication we find $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$.

Since every binomial may be put in the form $a+b$, we are able to write out its cube without the ordinary process of multiplication. Thus, $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$. Note the order of the signs.

Expand—

8. $(c+d)^3$.
9. $(c-d)^3$.
10. $(a+2b)^3$.
11. $(a-2b)^3$.
12. $(2a+b)^3$.
13. $(2a-b)^3$.
14. $(x+2)^3$.
15. $(x-2)^3$.
16. $(x-3)^3$.
17. $(x+4)^3$.
18. $(4+x)^3$.
19. $(4-x)^3$.
20. $(2x-1)^3$.
21. $(2x+2)^3$.
22. $(2x-3)^3$.
23. $(x+\frac{1}{2})^3$.
24. $(x-\frac{1}{2})^3$.
25. $(\frac{1}{4}+x)^3$.

Express the factors of—

26. $x^2+10xy+25y^2$.
27. $x^2+10xy+21y^2$.
28. $x^2+10xy+24y^2$.
29. $x^2+10xy+9y^2$.
30. $a^2+14ab+24b^2$.
31. $a^2+14ab+45b^2$.
32. $a^2+14ab+40b^2$.
33. $x^2+10xy-24y^2$.
34. $x^2-10xy-24y^2$.
35. $x^2+4xy-45y^2$.
36. $x^2-4xy-45y^2$.
37. $m^2+mn-42n^2$.
38. $m^2-mn-42n^2$.
39. $m^2-12m+35$.

Using x to represent the unknown number, write an algebraic expression to represent—

1. The number doubled.
2. The number to the third power.
3. The number increased by 5.
4. The number less one half the number.
5. The sum of 10 and twice the number.
6. The number increased by one half of itself.
7. The excess of the number over 100.
8. The excess of 100 over twice the number.
9. The next two consecutive smaller numbers.
10. The next two consecutive larger numbers.
11. The amount to be added to the number to equal 40.
12. The amount to be added to 40 to equal the number.
13. The sum of two numbers is 45. If x represents one of the numbers, what will represent the other?
14. The sum of two numbers is 73, and one of the numbers is 44. Find the other. What principle of arithmetic is illustrated by this problem?
15. The sum of two numbers is 120. One is 76. Find the other.
16. One person has \$10 more than another, and together they have \$40. How many dollars has each?
17. The difference of the ages of two persons is 14 years, and the age of the elder is 42 years. Find the age of the younger.
18. The sum of two numbers is 54, and their difference is 22. Find the numbers.
19. If x and $2x+6$ represent two numbers whose difference is 22, what are the numbers?
20. One boy has 22 marbles more than his brother, which number also equals 6 more than double as many as his brother has. Find how many each has.
21. One number is greater than another by 2, and their sum is 50. Find the greater number.
22. Divide 30 into two numbers whose difference is 6.

1. Divide 100 into two numbers whose difference is 10.

2. Divide \$300 among three persons, A, B, and C, so that B may receive twice as much, and C three times as much as A.

3. One number is less than another by 3, and their sum is 39. Find the numbers.

4. The sum of two numbers is 50, and one is greater than the other by 10. What are the numbers?

5. A father is 52 years old and his son is 4 years old. In how many years will the father be exactly 7 times as old as his son?

6. If 42 be added to a certain number, the result is 4 times that number. Find the number.

7. A person has \$630. Part of it he loans at the rate of 4%, and the remainder at the rate of 5%, and he received equal sums as interest from the two parts. How much did he loan at each rate?

8. Find a number such that its fifth part may exceed its seventh part by 12.

9. A father's age is 6 times as great as that of his son, but 4 years ago it was 11 times as great. Find the age of each.

10. John and Charles play a game of marbles. John has 22 marbles, and Charles 13 before they begin, and at the end of the game John has 4 times as many as Charles. How many did John win?

11. How much tea, worth 30 cents a pound, must be mixed with 12 pounds, at 50 cents a pound, to make a mixture worth 36 cents a pound?

Suggestion: 1 lb. at 30 cents gains —; 1 lb. at 48 cents loses —.

12. Divide \$480 into two parts, so that the first part put out at interest for a year, at 5%, may exceed the interest on the other part, at 6%, by \$20.70.

13. Find a number such that if 20 be added to it, the sum will be three times the remainder when 20 is subtracted from it.

14. Two boys have 74 marbles between them, and John has 46 more than James. How many marbles has each?

1. Divide 108 into two parts, so that 25% of one part may equal 20% of the other part.

SOLUTION.

Let (1) x = one part,

(2) $108-x$=the other part,

(3) 25 per cent of $x=\frac{x}{4}$,

(4) 20 per cent of $108-x=\frac{108-x}{5}$,

(5) $\frac{x}{4}=\frac{108-x}{5}$

Multiplying by 20, = (6) $5x = 432-4x$,

Transposing, (7) $9x = 432$,

Dividing, (8) $x = 48$, one part,

(9) $108-x = 60$, the other part.

2. What number subtracted from the denominator of $\frac{8}{9}$ will make the fraction equal to $\frac{3}{4}$?

3. Find the number whose $\frac{1}{2}$ and $\frac{1}{5}$ exceeds its $\frac{1}{3}$ and $\frac{2}{7}$ by 34.

4. What number subtracted from both numerator and denominator of $\frac{3}{4}$ will reduce the fraction to $\frac{1}{2}$?

5. A company took a risk at 4% and reinsured $\frac{3}{4}$ of it at 3%. The premium received exceeded the premium paid by $28. Find the amount of the risk?

6. What number added to the numerator and denominator of $\frac{15}{7}$ will make the fraction equal to 2?

7. What number added to the numerator and subtracted from the denominator of $\frac{5}{8}$ will make a fraction equal to $2\frac{1}{4}$?

8. A man lends $750 at a certain rate of interest, and $3,824 at a rate 2% higher. If the interest for one year from both investments is $95.78, find the rate at which each amount was loaned.

9. A farmer sold 50 acres more than one half of his farm, and still had 30 acres less than one third of it. How many acres were in the farm?

1. Find four consecutive numbers whose sum is 150.

2. Find two numbers whose sum is 156, and whose difference is 40.

3. Divide the number 320 into 2 parts, so that the greater shall exceed the less by 84.

4. Find a number as much larger than 20 as three times the number is greater than 70.

5. Divide 16 into two such parts that four times one part plus 5 times the other will equal 76.

6. If a number is doubled it will then be as much greater than 40 as three times the number is less than 85. Find the number.

7. One half the sum of two numbers is 12, one half of the difference is 3. Find the numbers.

8. Divide 72 into two such parts that three times one part will equal five times the other part.

9. Show that if the difference between two numbers is one, the difference of their squares is equal to the sum of the numbers. Show both by diagram and by Algebra.

10. What number is as much greater than 340 as one third of the number is greater than 34?

11. If a certain number is divided by 3, the sum of the divisor, dividend, and quotient will be 111. Find the number.

12. A young man is 18 years old, and his little sister is 4 years old. In how many years will his sister be two thirds of her brother's age?

13. A man has 5 sons with 3 years between the ages of any two successive ones, and the sum of all their ages is 10 times the age of the youngest. What is the age of each?

14. An amount of money is divided among several boys. If each boy takes 8 cents, there will be 5 cents remaining, but if each takes 9 cents the amount will be 5 cents too small. Find the number of boys.

15. A sixth of the sum of two numbers is 17, and $\frac{1}{4}$ their difference is 11. Find the numbers.

POWERS AND ROOTS.

Any number is the *first power* of itself.

If a number is multiplied by itself the product is called the *second power* of the number.

A number used repeatedly to form a power is called a *root* of that power. Thus, 2×2 is 4; the *power* is 4, the *root* is 2.

INVOLUTION is the process of finding powers.

If a number is used twice as a factor the product is called the second power, or square; if used three times, the product is called the third power, or cube.

Thus, $3 = 3$, first power, or root,
$3\times3 = 9$, second power, or square.
$3\times3\times3 = 27$, the third power, or cube.
$3\times3\times3\times3 = 81$, the fourth power.
$3\times3\times3\times3\times3 = 243$, the fifth power.

Powers are expressed by an index figure, or *exponent*, written at the right and above the root, to denote the number of times the root is used as a factor. Thus, $5^2 = 25$; $5^3 = 125$.

When a root of a number can be found, the number is said to be a *perfect power*.

EVOLUTION is the process of finding the root of a given number. It is the converse of involution.

The square root is indicated by the *radical sign* $\surd$. Any other root is indicated by a small figure called an *index*, which is placed a little above and at the left of the radical sign. When no index is written, the square root is required.

Thus, $\surd 49$ or $\sqrt[2]{49}$ means the square root of 49.
$\sqrt[4]{16}$ means the fourth root of 16.
$\sqrt[7]{4^6}$ means the seventh root of the sixth power of 4.

Roots may also be indicated by fractional exponents. Thus, $9^{\frac{1}{2}} = 3$. The denominator of the fractional exponent indicates the root, the numerator indicates the power. Thus, $4^{\frac{3}{2}} = \surd 4^3$, which expresses the square root of the third power of 4.

SQUARE ROOT.

Since $(a+b)^2 = a^2+2ab+b^2$, it is evident that any number of two figures may be represented by $a+b$, the tens being represented by a and the units by b. Or it may be better understood to represent a number of two figures by $t+u$. Thus,

$$(t+u)^2 = t^2+2tu+u^2.$$

By this formula,

$$(25)^2 = (20+5)^2 = (20)^2+(2\times20\times5)+5^2 = 625.$$
$$(32)^2 = (30+2)^2 = (30)^2+(2\times30\times2)+2^2 = 1024.$$

This may be illustrated by a diagram:

ab	b^2
a^2	ab
a	b

$(a+b)^2 = a^2+2ab+b^2$.

THE SQUARE ROOT of a number is a number taken twice as a factor to produce the given number.

The square of a number has twice as many figures as the root, or one less than twice as many. Thus,

Roots, 1, 10, 100.
Squares, 1, 100, 10000.

The square of any number less than 10 must be less than 100; but any number less than 10 is expressed by one figure, and any number less than 100 by less than three figures: therefore, the square of a number consisting of one figure is a number of either one or two figures.

The square of any number between 10 and 100 must be between 100 and 10,000; that is, must contain more than two figures and less than five.

Hence, to ascertain the number of figures in the square root of a given number: Beginning at units, point off the number into periods of two places each by placing a dot over the first or unit figure, and one over each second figure towards the left. There will be as many figures in the root as there are dots.

1. Find the square of $3ax^2$.
2. Find the square of $5b^2c^3$.
3. Find the cube of $2x^2y^3$.
4. Find the square of $-ab^2$.
5. Find the cube of $-abc^2$.
6. Find the fourth power of $3ab^3$.

1. Find the square root of 1,296.

SOLUTION.

$t^2+2tu+u^2 = 12\dot{9}6\ (30$

$t^2 = 900 \quad 6$

$2tu+u^2 = 396 \quad 36$

$2tu = 360$

36

$u^2 = 36$

The greatest number of tens whose square can be found in 12 hundreds is 3 tens, which we write in the quotient. Subtracting the square, 900, the remainder is 396 = twice the tens times the units, plus the square of the units. To find the units' figure, by way of trial, divide 396 by twice the tens, or 60. The quotient is 6. By completing the formula, it is found that there is no remainder, and 36 is the required square root.

2. A square lot contains 1,296 square rods. What is the length of one side?

SOLUTION.

$12\dot{9}6\ (30$

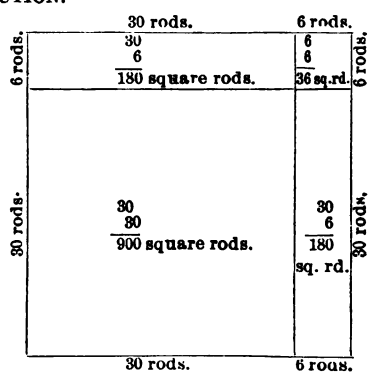
$30\times30 = 900 \quad 6$

$396 \quad 36$

$2\times30\times6 = 360$

36

$6\times6 = 36$



Find the square root of—

3. $4a^2x^2$.

4. $9x^2y^4$.

5. $25a^2b^2c^4$.

6. $36a^4b^6x^2$.

7. x^2+4x+4.

8. $4x^2-12x+9$.

9. $x^2y^2-8xy+16$.

10. $4a^2x^2+25y^2z^2-20axyz$.

11. $\frac{4a^2}{9b^2}$.

12. $\frac{16x^2y^4}{25a^2z^2}$.

13. 961.

14. 1369.

CUBE ROOT.

Since $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$, it is evident that the third power of a number of two figures may be found by using this formula. The number 25 equals 20+5.

$$(25)^3 = (20+5)^3$$

$$(20+5)^3 = \begin{cases} (20)^3 = 8000 \\ 3\times(20)^2\times 5 = 6000 \\ 3\times 20\times 5^2 = 1500 \\ 5^3 = 125 \\ \hline 15625 \end{cases}$$

Find the cube of—

1.	12.	**5.**	32.	**9.**	45.	**13.**	44.
2.	13.	**6.**	25.	**10.**	17.	**14.**	27.
3.	14.	**7.**	31.	**11.**	26.	**15.**	35.
4.	21.	**8.**	32.	**12.**	33.	**16.**	43.

THE CUBE ROOT of a number is one of its three equal factors.

Thus, 2 is the cube root of 8; for $2\times2\times2 = 8$; and 3 is the cube root of 27, for $3\times3\times3 = 27$.

To extract the cube root of a number is to find one of its three equal factors.

Digits:	1	2	3	4	5	6	7	8	9
Cubes:	1	8	27	64	125	216	343	512	729

A PERFECT CUBE is a number composed of three equal factors. By examining the numbers in the two lines we see—

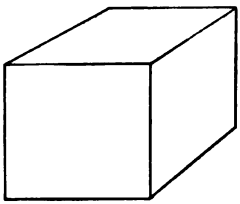
That the cube of units cannot be given a higher order than hundreds.

That the cube of tens will not give a lower order than thousands, nor a higher denomination than hundreds of thousands.

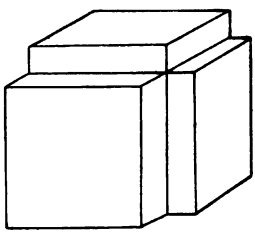
Hence, if a number contains more than three figures, its cube root will contain more than one; if it contains more than six, its root will contain more than two, and so on. Every additional three figures gives one additional figure in the root, and the figures which remain at the left hand, although less than three, will also give a figure in the root.

1. Find the edge of a cube containing 91,125 cubic inches.

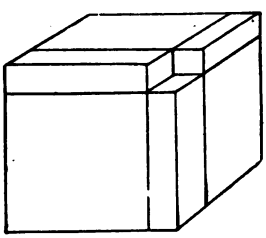
SOLUTION.



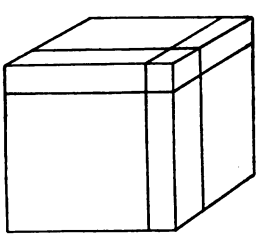
The largest cube readily determined by inspection is $40^3 = 40 \times 40 \times 40 =$ 64,000 cubic inches. Subtracting, 91,125 —64,000 = 27,125 cubic inches, remaining.



This consists chiefly of three equal faces or added blocks, having a surface of $3 \times 40^2 = 3 \times 40 \times 40 =$ 4,800 square inches of surface. The thickness of these added blocks is determined by inspection to be 5 inches; hence, they contain $5 \times$ 4,800 = 24,000 cubic inches. Subtracting, 27,125—24,000 = 3,125 cubic inches remaining.



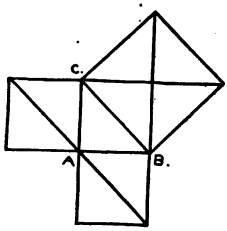
This remainder consists of 3 corner pieces each $40 \times 5 \times 5$, containing 3,000 cubic inches; also, a 5-inch cube, containing 125 cubic inches, which completes the cube.



The work is usually written in the following form:

$$
\begin{array}{rrrr}
 & & 91125 & (40 \\
40^3 = & & 64000 & \underline{\ \ 5} \\
3 \times 40^2 = & 4800 & 27125 & 45 \\
3 \times 40 \times 5 = & 600 & & \\
5^2 = & \underline{\ 25} & & \\
 & 5425 \times 5 = & \underline{27125} &
\end{array}
$$

APPLICATIONS OF SQUARE AND CUBE ROOT.



The triangle A B C is formed by drawing B C, the diagonal of a square. By making a square on each of the three sides of the triangle, and drawing diagonals through each square, as shown in the figure.

1. How many triangles are formed, not counting the triangle A B C?

2. How many in the large square?

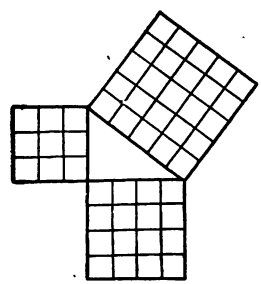
3. How many in each of the smaller squares?

4. Are they all of equal size?

5. Is the square on B C equal to the sum of the squares on A B and A C?

It is proved in Geometry—

(1). That the square described on the hypotenuse of any right-angled triangle is equal to the *sum of the squares* described on the other two sides.



(2). That the square on either of the sides forming the right angle is equal to the square on the hypotenuse *diminished by the square* on the other side.

Hence,

To find the hypotenuse—

Take the square root of the sum of the squares of the other two sides.

To find the base or perpendicular—

Take the square root of the difference of the squares of the hypotenuse and the other side.

Let H = hypotenuse, B = base, and P = perpendicular:

$$H = \sqrt{B^2 + P^2}.$$

$$B = \sqrt{H^2 - P^2}.$$

$$P = \sqrt{H^2 - B^2}.$$

1. The two sides of a right-angled triangle are 3 and 4 feet. What is the length of the hypotenuse?

SOLUTION.

$3^2 = 9$, the square on one side,
$4^2 = 16$, the square on the other side,
25, the square on the hypotenuse,
$\sqrt{25} = 5$, one side of the square, or the hypotenuse.

2. The floor of a room is 6 feet wide and 8 feet long. What is the diagonal distance from one corner to the opposite corner?

SOLUTION.

Length squared $(6^2) = 36$ sq. ft.,
Width squared $(8^2) = 64$ sq. ft.,
$36+64 = 100$ sq. ft.,
Diagonal $= \sqrt{100} = 10$ sq. ft.

3. The distance from the top of a post 8 feet high to the end of its shadow is 10 feet. Find the length of the shadow.

SOLUTION.

Square of the hypotenuse $(10^2) = 100$,
Square of the perpendicular $(8^2) = 64$,
Subtract to find the square of the base $= 36$,
Base $= \sqrt{36} = 6$ feet.

NOTE.—It is seen from these solutions that the sides of a right-angled triangle may readily be computed if they are in the ratio of 3, 4, and 5.

4. How long must a ladder be to reach the top of a house 40 feet high, when the foot of it is 30 feet from the house?

5. Suppose a ladder 100 feet long to be placed 60 feet from the roots of a tree. How far up the tree will the top of the ladder reach?

6. Two persons start from the same place and go, the one due north 60 miles, the other due west 80 miles. How far apart are they?

7. A flagstaff was broken $\frac{1}{3}$ of the way up, and the top struck the ground 27 feet from the foot. How high was the flagstaff?

1. A rope 45 feet long, attached to the top of a telegraph pole, extended to the ground, 36 feet from its base. How high was the pole?

2. A man has 200 yards of carpeting $1\frac{1}{3}$ yards wide. Find the length in feet of one side of the square room which his carpet will cover.

3. There is a wall 45 feet high, built upon the bank of a stream 60 feet wide. How long must a ladder be that will reach from one side of the stream to the top of the wall on the other?

4. Find the height of a pole, which, being broken 39 feet from the top, the end will strike the ground 15 feet from the foot.

5. The top of a castle is 45 yards high, and the castle is surrounded by a ditch 60 yards wide. Find the length of a rope that will reach from the outside of the ditch to the top of the castle.

6. Find rods of fence required to enclose 10 acres of land in the form of a square.

7. How much more will it cost to fence 10 acres of land, in the form of a rectangle, the length which is four times its breadth, than if it were in a form of a square, the cost of the fence being $2.50 a rod?

8. A ladder 52 feet long stands close against the side of a building. How many feet must it be drawn out at the bottom that the top may be lowered 4 feet?

9. A flagpole planted in a street is 50 feet high. If it should fall directly toward one side of the street the top would strike the building 40 feet from the ground. If it should fall in the opposite direction it would strike the building 30 feet from the ground. How wide is the street?

10. Show by cutting paper how to construct a square equal in area to the area of two given squares.

11. Show by cutting paper how to construct a square equal in area to the difference between the areas of two given squares.

12. Find the approximate length of one side of a square whose diagonal is 10 feet.

1. Find the number of which the product of its third and fourth parts is equal to 108.

2. Find the number whose square diminished by 16, is equal to half its square increased by 16.

3. Find the number whose square diminished by 54, is equal to the square of its half increased by 54.

4. Find the surface of a cubical block containing 15,625 cubic inches.

5. Find the approximate length of the side of a cube which will contain as much as a box 8 feet 3 inches long, 3 feet wide, and 2 feet 7 inches deep.

6. Find the number, which being divided by 9, gives the same quotient as 16 divided by the number.

7. If a cubical body contains 6,859 cubic feet, what is the length of one side? What is the area of its surface?

8. Find two numbers that are to each other as 3 to 5, and the difference of whose squares is 64.

Let $3x$ = the less number, $5x$ = the greater.

9. Find two numbers that are to each other as 3 to 4, and the difference of whose squares is 63.

10. Find two numbers that are to each other as 3 to 4, and the sum of whose squares is equal to 100.

11. Find a number, to which if 3 be added, and from which if 3 be subtracted, the product of the sum and difference shall be equal to 40.

12. The width of a lot of ground is to its length, as 5 to 9, and it contains 1,620 square feet. Find the dimensions.

13. A man bought a number of oranges for 48 cents, and the price of an orange was to the number bought, as 1 to 3. How many did he buy, and what was the price?

14. Find the contents of a cube having 1,944 square inches of surface.

15. A square piece of timber contains 27 cubic feet, and is 8 times its thickness or width. What are its dimensions?

Suggestion: Cut crosswise into 8 equal parts. Each is a cube.

SHORT METHODS.

By using multipliers which are larger by 1, or smaller by 1, than 10, $12\frac{1}{2}$, $16\frac{2}{3}$, 20, 25, $33\frac{1}{3}$, 50, 100, etc., the operation may be shortened. Thus, to multiply a number—

By 11 , multiply by 10 , and add the number.
By 9 , multiply by 10 , and subtract the number.
By $13\frac{1}{2}$, multiply by $12\frac{1}{2}$, and add the number.
By $11\frac{1}{2}$, multiply by $12\frac{1}{2}$, and subtract the number.
By $17\frac{2}{3}$, multiply by $16\frac{2}{3}$, and add the number.
By $15\frac{2}{3}$, multiply by $16\frac{2}{3}$, and subtract the number, etc.

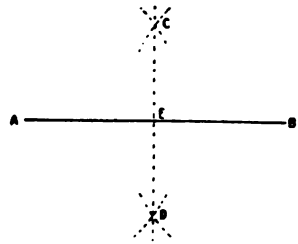
1.	$24\times 9 =$	**21.**	$23\times 9 =$
2.	$24\times 11 =$	**22.**	$23\times 19 =$
3.	$24\times 11\frac{1}{2} =$	**23.**	$40\times 11\frac{1}{2} =$
4.	$24\times 13\frac{1}{2} =$	**24.**	$40\times 13\frac{1}{2} =$
5.	$24\times 15\frac{2}{3} =$	**25.**	$40\times 112\frac{1}{2} =$
6.	$24\times 17\frac{2}{3} =$	**26.**	$40\times 111\frac{1}{2} =$
7.	$35\times 19 =$	**27.**	$40\times 113\frac{1}{2} =$
8.	$35\times 21 =$	**28.**	$32\times 125 =$
9.	$36\times 24 =$	**29.**	$32\times 124 =$
10.	$36\times 26 =$	**30.**	$32\times 126 =$
11.	$36\times 32\frac{1}{3} =$	**31.**	$36\times 116\frac{2}{3} =$
12.	$36\times 34\frac{1}{3} =$	**32.**	$36\times 115\frac{2}{3} =$
13.	$36\times 49 =$	**33.**	$36\times 117\frac{2}{3} =$
14.	$36\times 51 =$	**34.**	$35\times 120 =$
15.	$36\times 99 =$	**35.**	$35\times 119 =$
16.	$36\times 15\frac{2}{3} =$	**36.**	$35\times 121 =$
17.	$36\times 17\frac{2}{3} =$	**37.**	$36\times 133\frac{1}{3} =$
18.	$36\times 11\frac{1}{2} =$	**38.**	$36\times 132\frac{1}{3} =$
19.	$36\times 13\frac{1}{2} =$	**39.**	$36\times 134\frac{1}{3} =$
20.	$72\times 99 =$	**40.**	$36\times 151 =$

41. Find the cost of 80 acres at $\$112\frac{1}{2}$ per acre.
42. Find the cost of 80 acres at $\$111\frac{1}{2}$ per acre.
43. Find the cost of 80 acres at $\$113\frac{1}{2}$ per acre.
44. Find the cost of 90 acres at $\$134\frac{2}{3}$ per acre.

GEOMETRY AND MENSURATION.

PROBLEMS FOR CONSTRUCTION.

PROBLEM I. *To bisect a given straight line.*

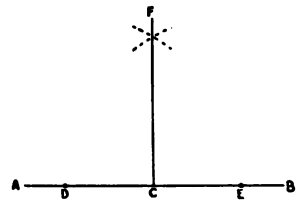


Let A B be the given straight line.

From A and B as centres, with equal radii, describe arcs intersecting at C and D, above and below the given line.

Draw the straight line C D. This line bisects the line A B.

PROBLEM II. *To erect a perpendicular at a given point in a given straight line.*



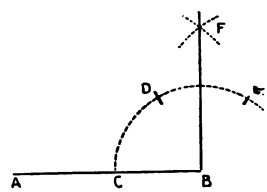
Let A B be the given line, and O be the given point.

Mark equally distant from O the points C and D in the given straight line.

From C and D as centres, with equal radii, describe arcs intersecting at F.

Draw the line F O. This line is the perpendicular desired.

PROBLEM III. *To erect a perpendicular at the end of a given straight line.*



Let A B be the given line.

With B as a centre describe the arc C E.

With C as a centre, and C B the radius, intersect the arc at D.

With D as a centre, and C B the radius, intersect the arc at E.

With D and E as centres, and with equal radii, draw arcs intersecting at F.

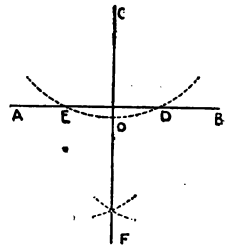
Draw the line B F. It is the perpendicular desired.

PROBLEM IV. *From a given point, draw a perpendicular to a given straight line.*

Let A B be the given straight line, and C the given point. With C as a centre, draw the arc intersecting A B at the points D and E.

With D and E as centres, draw the arcs intersecting at F.

Draw C F. It is the required perpendicular.



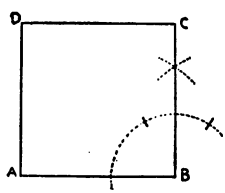
PROBLEM V. *The side of a square being given, to construct the square.*

Let A B be the given line.

At B erect a perpendicular to C equal to A B.

With A and C as centres, and a radius equal to A B, draw arcs intersecting at D.

Draw A D and C D to complete the square.



PROBLEM VI. *To bisect a given angle.*

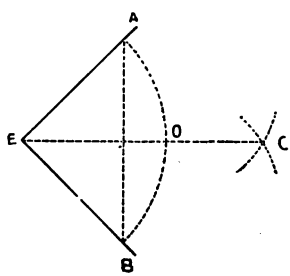
Let A E B be the given angle.

From E as a centre, with any radius, as E A, describe the arc A O B, terminating in the sides of the angle.

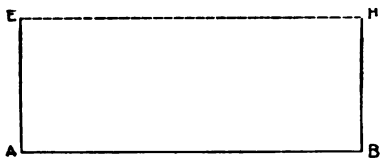
Draw the chord A. B.

From A and B as centres, with equal radii, describe two arcs intersecting at C.

Draw E C. It will bisect the angle E.



PROBLEM VII. *Through a given point to draw a straight line parallel to a given straight line.*



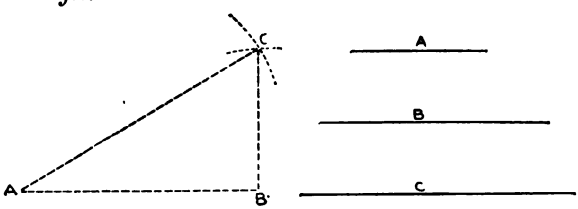
Let A B be the given line, and H the given point.

Draw the perpendicular B H.

At A draw the perpendicular A E equal to B H.

Draw the line E H. It is parallel to A B.

PROBLEM VIII. *The three sides of a triangle being given, to construct the triangle.*



Let the three sides be A, B, and C.

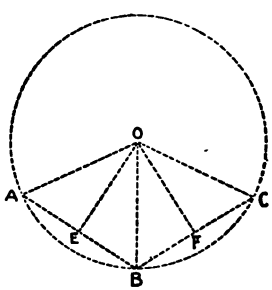
Draw A B equal to B.

From A as a centre, with a radius equal to C, describe an arc.

From B as a centre, with a radius equal to A, describe an arc intersecting the former arc at C.

Draw C A and C B. The triangle C A B is the one required.

PROBLEM IX. *To describe a circumference through three points not in the same straight line.*



Let A, B, and C be the three points.

Draw the straight lines A B and B C.

Bisect A B and B C.

At the points of bisection, E and F, erect perpendiculars intersecting at O.

From O as a centre, with a radius equal to O A, describe the circle.

DEFINITIONS AND PROBLEMS.

The following definitions and problems should be illustrated by figures drawn on the blackboard, and also by figures drawn on paper and cut out. *Consult the dictionary for exact definitions.* The pupil should be provided with a protractor and compasses.

1. Define a *Point.* Can you make a point on paper?
2. Define a *Line;* a straight line; a curved line.
3. Define a *Plane* surface. Define a *Solid.*
4. Define an *Angle; Right Angle; Perpendicular.*
5. Define *Horizontal Lines; Parallel Lines; Oblique Lines.*
6. Distinguish between a *Vertical* and a *Perpendicular* line.
7. Define *Rectangle; Base; Altitude; Diagonal.*
8. Define *Square; Cube.*
9. Distinguish between a *Solid* and *Capacity.*
10. Define *Circle; Circumference; Arc.*
11. Define *Radius; Diameter; Chord.*
12. Define a *Quadrilateral;* a *Parallelogram.*
13. Define *Rhombus; Rhomboid; Trapezoid.*
14. Define *Triangle; Base,* and *Altitude* of a triangle.
15. Define *Equilateral, Isosceles,* and *Scalene* triangle.
16. Define *Right-Angled Triangle* and *Hypotenuse.*
17. Define *Trapezium; Diagonal.*
18. Define *Degree; Sextant; Quadrant.*
19. Define a *Unit;* a *Measure.*
20. Name the standard unit of *Length;* of *Capacity;* of *Weight;* of the *Metric System.*
21. Define and draw an *Oblique* angle; *Acute* angle; *Obtuse* angle.
22. Use a protractor and draw angles of 90°, 45°, 30°, 60°, 75°, 120°, 135°.
23. Draw an angle of any size, and with the compasses draw another of the same size.
24. Draw a straight line, and draw another obliquely to the first. How many angles are formed? Angles formed by such a meeting of straight lines are called *Adjacent* angles.

25. Draw two straight lines crossing each other. How many angles are formed? How many are of the same size? The angles opposite each other are called *Vertical* angles.

26. Define *Vertical* angles; *Adjacent* angles.

27. How many degrees in the sum of two adjacent angles? Prove it.

28. If one of two adjacent angles is given, how find the other?

29. If one of four angles, formed by two straight lines crossing, is given, how find the others?

30. How small may be an acute angle? An obtuse angle?

31. How large may be an acute angle? An obtuse angle?

32. Draw a triangle with a base of 6 inches, and the angles at each end each 90°. If two angles of a triangle are each 90°, what is their sum? Can you construct a triangle if two of its triangles make 180° or more?

Construct on any given line—

33. A triangle, with two angles 45° and 45°.

34. A triangle, with two angles 30° and 90°.

35. A triangle, with two angles 90° and 45°.

36. A triangle, with two angles 60° and 60°.

37. A triangle, with two angles 60° and 45°.

38. How many straight lines can be drawn through two given points? How many can be drawn through one given point?

39. At one end of a 4-inch line erect a perpendicular 3-inch line. Measure the hypotenuse.

40. Use the ruler and pencil, and try to construct a triangle having sides, 6 inches, 2 inches, and 4 inches in length. Is it possible? Why?

41. Draw a line 6 inches long. With one end as a center, draw an arc, using a 6-inch string as a radius. With the other end as a center, draw another arc with the same radius, to intersect the first arc. Join the point of intersection with the ends of the line. What is the resulting figure? Are the sides equal?

42. Define an *Equilateral Triangle.*

43. Draw an equilateral triangle. From one of the angles draw a line to the middle of the opposite side. Fold the figure on this line. Two angles of the triangle are thus brought together. Do they coincide? In a similar manner draw from each angle and fold. *The angles of an equilateral triangle are equal.*

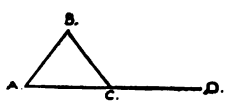
44. How many degrees in each angle of an equilateral triangle?

45. Draw two equal lines from the same point, making any angle. Join the other ends. What kind of triangle is formed? Fold the triangle so that the equal sides coincide. *The angles opposite the equal sides of a triangle are equal.*

46. Draw two straight lines to a point in a straight line, AB, making three angles on the same side of the straight line, AB. What is the sum of the three angles? What Axiom is illustrated?

47. Draw and cut out a triangle. Cut off two of the corners and place them by the sides of the other corner, with the three vertices at one point. Do they form a straight line? How many degrees on one side of a straight line? How many right angles? *The sum of the three angles of a triangle is equal to two right angles.*

48. Draw a rectangle. How many right angles within it? Draw a diagonal dividing the rectangle into two equal triangles. What part of the right angles within the rectangle is within each triangle? How many right angles in each triangle? The sum of the three angles of each triangle equals how many right angles? One of the angles is a right angle; the sum of the other two equals what?



49. Let ABC be a triangle. Extend the line AC to D. The angle BCD is called an *exterior* angle.

The angles ACB+DCB equal two right angles. Why?

The angles ACB+BAC+ABC equal two right angles. Why?

ACB+DCB=ACB+BAC+ABC. Why?

DCB= BAC+ABC. Why?

50. Show that the area of a triangle is equal to the base multiplied by one half of the altitude.

51. Draw a line AB, 8 inches long. With A as a center, draw an arc with a radius of 7 inches. With B as a center, draw an arc intersecting the first with a radius of 9 inches. In like manner, on a line 7 inches long complete a triangle having the other two sides 8 inches and 9 inches long; and on a line 9 inches long complete a triangle having the other two sides 7 inches and 8 inches long. Cut out the three triangles. Are they of the same size? *Two triangles are equal when the three sides of one are respectively equal to the three sides of the other.*

52. Draw a trapezium. Divide it into two triangles by a diagonal. The angles of each triangle equal what? The angles of the trapezium equal what? *The sum of all the angles of a quadrilateral is equal to four right angles.*

53. Draw a trapezoid. Prove that the area of a trapezoid equals the product of its altitude and one half the sum of its parallel sides.

54. Draw a parallelogram. Divide it into two triangles by a diagonal. Cut out the triangles. Are they equal in size? Are they of the same form? Show that *the opposite angles of a parallelogram are equal.*

55. How find the area of a parallelogram?

56. Define a *Regular Polygon.* Draw one on the board.

57. Define a *Regular Pentagon.* Draw one on the board.

58. Define a *Regular Hexagon.* Draw one on the board.

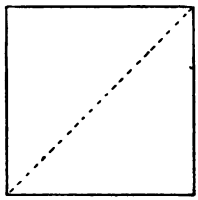
59. Define a *Regular Septagon.* Draw one on the board.

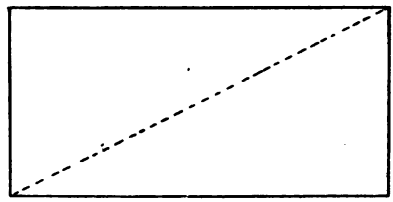
60. Define a *Regular Octagon.* Draw one on the board.

61. Draw six equal equilateral triangles about a common point. Can you draw a circumference touching the six outside vertices?

62. How find the area of a regular polygon or circle? Illustrate by drawing on the board.

63. Divide a parallelogram into two equal parallelograms. Into four equal parallelograms. How did you do it?





64. Draw and cut out a triangle having the area of a given parallelogram.

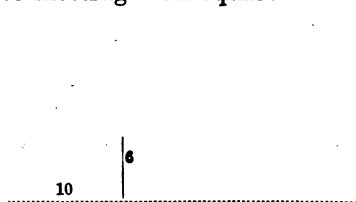
65. Draw a triangle having four times the area of a given similar triangle. Nine times the area.

66. Draw and cut out a triangle having the area of a given square. Suggestion: Form a rectangle by placing two squares together. Draw a diagonal. Or, cut a square along the diagonal, and arrange the two parts in the form of a triangle.

67. From a point draw five equal lines making five equal angles at the given point. Connect the extremities of adjacent lines by straight lines. Are the connecting lines equal?

68. A post, 6 feet high, casts a shadow 10 feet long. How find the height of a tree by measuring the length of its shadow?

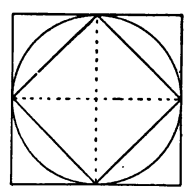
NOTE.—In similar triangles the corresponding sides are proportional.



69. Show by the same method how to find the distance to a tree across a river, without measuring across.

70. Define a *Tangent.* Draw one on the board.

71. Draw a square the sides of which are tangent to a circumference, and a square the corners of which touches the same circumference. Compare their areas.



72. Draw a square equal in area to the sum of a three-inch square and a four-inch square.

SOLUTION.

Draw a four-inch line.

Draw a perpendicular at one end three inches high.

Draw a hypotenuse.

Draw a square, one side of which equals this hypotenuse.

73. Draw two squares, not of the same size. Make a third square having an area equal to the sum of the areas of the other two.

74. Define and draw a *Chord; Secant; Sector; Segment.*

75. Find the centre of a given circle.

NOTE.—Draw two chords; draw perpendiculars from their middle points.

76. Draw two circles tangent to each other.

NOTE.—Draw a straight line, and locate both centres in this line.

77. Inscribe a circle in a triangle.

NOTE.—Draw lines bisecting the angles of the triangle to locate the centre.

78. Circumscribe a circle about a triangle.

NOTE.—Bisect the sides of the triangle to locate the centre.

79. Show how to find the area of a *Sector.*

80. On a given straight line, construct a isosceles triangle whose equal sides are equal to a given straight line.

81. Show by cutting into triangles that the diagonals of a parallelogram bisect each other.

82. Find the point in the base of a triangle which is equidistant from the other two sides.

83. Draw a quadrilateral, the diagonal of which is 2 feet, and the sides are 16, 12, 20, and 15 inches, respectively.

84. If one angle of a triangle equals the sum of the other two, the triangle can be divided into two isosceles triangles. Prove by cutting paper.

85. Construct a rhombus, the diagonal and the length of one side being given.

86. Cut a rhombus by a straight line so as to form a rectangle of the two parts.

87. Cut a triangle by a straight line so as to form a parallelogram of the two parts.

PARTIAL PAYMENTS.

1. If a man borrows \$1,000, on what sum does he pay interest? If he pays part of the debt, should he still pay interest on \$1,000? On how much should he pay interest after he pays part of the debt? For what time should he pay interest on the whole debt? For how long should he pay interest on the remaining debt?

2. A man borrowed \$100 for 2 years, at 10%. He paid \$60 at the end of the first year. How much was due at maturity?

SOLUTION.

Principal drawing interest	\$100
Interest for one year	10
Amount due at end of first year	110
Payment at end of first year	60
Amount unpaid drawing interest to maturity	50
Interest for one year	5
Amount due at maturity	55

3. A merchant borrowed \$200 for 18 months, at 6%, but paid \$112 at the end of one year. How much was due at maturity?

4. A grocer gave to a wholesale firm a note for \$500, due in 9 months, bearing 12% interest. At the end of 6 months he paid \$130. How much was due at the end of the nine months?

A *Partial Payment* is a payment of only a part of a debt.

When a partial payment of a note is made it is *indorsed*, or written on the back of the note.

An *indorsement* states the date and amount of the payment.

UNITED STATES RULE FOR PARTIAL PAYMENTS.

Find the amount of the principal to the time when a payment or the sum of two or more payments equals or exceeds the interest. From this amount deduct the payment or the sum of the payments.

With the remainder as a new principal, proceed as before.

NOTE.—**This rule is founded on the principle that neither interest nor payment shall draw interest.**

1.

$500.00. CHICAGO, ILL., May 1, 1896.

On demand I promise to pay to L. VanOsdell, or order, five hundred dollars, with interest at 6%, for value received.

J. A. SMITH.

On the back of this note is written:

September 1, 1896, paid $ 50.00.
March 1, 1897, paid..... 15.00.
May 1, 1897, paid..... 163.40.

What amount is due November 1, 1897?

SOLUTION.

Principal	$500.00
Interest to time of first payment (4 months)	10.00
Amount due	$510.00
Payment	50.00
New principal	$460.00

Interest to the next payment (6 months) is $17.80, which is more than the payment. If the payment is subtracted as before, $2.80 of the interest will become part of the principal, and will draw interest. But interest on interest (compound interest) is not legal. Therefore, the interest is found to the time of the third payment (8 months).. 18.40

Amount due	$478.40
Sum of second and third payments	178.40
New principal	$300.00
Interest to date of settlement	9.00
Amount due November 1, 1897	$309.00

2. April 10, 1898, a note was given for $1,680, at 6% interest. Payments were made January 19, 1899, of $185; and November 3, 1899, of $30. What was due January 5, 1900?

3. A note dated May 1, 1879, for $4,000, on interest at 6%, had following payments indorsed: May 21, 1880, $800; June 10, 1881, $1,200; August 10, 1882, $1,500. What was due May 1, 1883?

BANKING.

A CORPORATION is a collection of persons authorized by law to do business together. The instrument which defines their rights and powers is called a *Charter*.

BANKS are corporations for the purpose of receiving deposits, loaning money, and furnishing a paper circulation represented by specie.

The sum charged by a bank for cashing a note or time draft is called *bank discount*.

This *discount* is the simple interest, paid in advance, for the number of days the note or draft has to run.

When a note is discounted at a bank the *payee indorses it*, making it payable to the bank. Both maker and payee are then responsible to the bank for its payment.

The *proceeds* of a note is the amount on which the discount is reckoned, less the discount.

If the note is drawing interest the discount will be reckoned on and deducted from the *amount due at maturity*.

Most notes discounted at banks do not draw interest.

The *time* in *bank discount* is always the number of days from the *date of discounting* to the *date of maturity*.

If a merchant desires to borrow, on April 10, $1,000 for 60 days, he may take his own note, or the note of another party, for $1,000, to a bank, and if the note is properly indorsed, or its payment is otherwise secured, the bank will take it, pay him $1,000, less the interest for 60 days, and collect the $1,000 when it becomes due.

The interest on $1,000 for 60 days is $10.00. $1,000—$10.00 = $990, the sum which the merchant will receive on the note.

The $1,000 is the *face* of the note. The $10.00 is the *discount*. The $990 is the *proceeds*. The 60 days is the *time*.

The note is due 60 days after April 10, or June 9. In some states it is legally due 3 days later, June 12, which is the date of *maturity*. These dates are written June 9/12. The three additional days granted by law are called days of grace.

1. A note dated July 9 is payable in 60 days. What is the date of maturity?

SOLUTION.

60 days = time to run,
22 days remaining in July,
38 days after July,
31 days in August,
7 days after August = September 7, date of maturity.

NOTE.—When the time is given in days, the exact days are to be reckoned. Add three days of grace, if required by law.

Find the date of maturity—

2. Of a note dated August 3, time 15 days.
3. Of a note dated July 4, time 20 days.
4. Of a note dated May 9, time 30 days.
5. Of a note dated June 3, time 35 days.
6. Of a note dated April 13, time 45 days.
7. Of a note dated July 9, time 60 days.
8. Of a note dated June 16, time 18 days.
9. Of a note dated May 15, time 40 days.
10. Of a note dated January 9, time 34 days.
11. Of a note dated July 5, time 30 days.
12. Of a note dated October 12, time 60 days.
13. Of a note dated December 23, time 90 days.
14. Of a note dated January 24, time 70 days.
15. Of a note dated February 18, time 65 days.
16. Of a note dated September 19, time 45 days.
17. Of a note dated April 30, time 85 days.
18. Of a note dated November 21, time 47 days.
19. Of a note dated March 13, time 35 days.
20. Of a note dated December 10, time 90 days.
21. Of a note dated April 19, time 20 days.
22. Of a note dated April 15, time 60 days.
23. Of a note dated January 15, time 90 days.
24. Of a note dated December 20, time 90 days.

1. A note is dated January 5, payable in 3 months. Find the date of maturity and days to run.

SOLUTION.

3 months after January 5 is April 5,
26 days remaining in January,
28 days remaining in February,
31 days remaining in March,
5 days remaining in April,
90 days to run.

NOTE.—When the time is given in months, reckon even months. Add 3 days of grace if required by law.

Find the date of maturity—

2. Of a note dated January 5, time 9 months.
3. Of a note dated February 3, time 8 months.
4. Of a note dated December 7, time 4 months.
5. Of a note dated July 16, time 3 months.
6. Of a note dated May 23, time 6 months.
7. Of a note dated June 27, time 4 months.
8. Of a note dated November 9, time 3 months.
9. Of a note dated September 3, time 2 months.
10. Of a note dated October 10, time 4 months.
11. Of a note dated January 8, time 5 months.
12. Of a note dated September 7, time 10 months.
13. Of a note dated November 9, time 11 months.
14. Of a note dated December 1, time 13 months.
15. Of a note dated April 15, time 2 months.
16. Of a note dated March 22, time 3 months.
17. Of a note dated May 20, time 9 months.
18. Of a note dated June 18, time 5 months.
19. Of a note dated July 29, time 6 months.
20. Of a note dated April 30, time 8 months.
21. Of a note dated May 30, time 5 months.
22. Of a note dated December 20, time 3 months.
23. Of a note dated August 15, time 2 months.

Find the number of days from the date of discount to the date of maturity of a note—

1. Dated Jan. 18, time 3 months, discounted Jan. 18.
2. Dated Nov. 21, time 4 months, discounted Nov. 21.
3. Dated Dec. 13, time 5 months, discounted Jan. 13.
4. Dated Mar. 14, time 2 months, discounted Mar. 18.
5. Dated Aug. 22, time 3 months, discounted Sept. 14.
6. Dated May 30, time 30 days, discounted June 15.
7. Dated Jan. 28, time 30 days, discounted Jan. 31.
8. Dated Nov. 14, time 60 days, discounted Nov. 20.
9. Dated July 12, time 45 days, discounted July 20.
10. Dated Sept. 19, time 90 days. discounted Oct. 10.

What are the bank discount and the proceeds—

11. Of a note for $600, due in 21 days, discounted at 6%.

SOLUTION.

Face of the note = $600.00,
Interest for 21 days = 2.10,
Proceeds (subtracting) = $597.90.

NOTE.—If the law requires days of grace, find interest for 24 days.

12. Of a 4-month note for $1,750, at 6 per cent.
13. Of a 6-month note for $ 428, at 7 per cent.
14. Of a 90-day note for $1,200 dated January 4, 1891, and discounted February 3, 1891, at 6 per cent.
15. Of a 4-month note for $10,000, discounted 15 days after date, at 8 per cent.
16. Of a 6-month note for $5,250, discounted 60 days after date, at 5 per cent.
17. Of $240 from May 30 to Nov. 26, 1895, at 10 per cent.
18. Of $334 from Aug. 2 to Nov. 30, 1896, at 5 per cent.
19. Of $672 from Feb. 28 to Oct. 25, 1899, at 4½ per cent.
20. Of $ 60 from June 20 to Nov. 10, 1899, at 12 per cent.
21. Of $600 from July 4 to Oct. 20, 1892, at 3 per cent.
22. Of $630 from Feb. 3 to Aug. 22, 1893, at 5½ per cent
23. Of $480 from Jan. 22 to Dec. 3, 1899, at 5 per cent
24. Of $270 from May 11 to July 30, 1894, at 6 per cent.

1. What are proceeds of a note for $10,000, payable in 6 months, and bearing interest at 6 per cent, if discounted 2 months before its maturity, at 6 per cent?

NOTE.—When a note bearing interest is discounted before its maturity, the *amount* of the note *at maturity*, rather than its *face*, is the base for discounting.

SOLUTION.

Face of the note	=	$1000.00
Interest to maturity (6 mo.)	=	30.00
Amount to be discounted	=	$1030.00
Discount for 2 mo. at 6 per cent	=	10.30
Proceeds (subtracting)	=	$1019.70

2. Find the bank discount and the proceeds of a note for $3,000, dated May 10, 1900, payable in 6 months, with interest at 8 per cent, and discounted Nov. 25, 1900, at the same rate.

3. Find the proceeds of a note for $1,500, due in 90 days, bearing interest at 5 per cent, and discounted at a bank at the same rate.

4. A merchant wishes to get $800 from a bank. For what amount must he give his note for 30 days, discount being 6 per cent?

NOTE.—Since he receives only the *proceeds*, the *face* of the note will be found by dividing the given proceeds by the proceeds of $1 for the given rate and time.

SOLUTION.

On a face of $1.00,
The discount = .005,
The proceeds = .995,
$800 ÷ $.995 = $804.02, the face of the note.

Find the face of a note, the proceeds of which—

5. Are $1944 for 4 months 24 days, at 7 per cent.
6. Are $ 389 for 6 months 18 days, at 5 per cent.
7. Are $4880 for 3 months 18 days, at 8 per cent.
8. Are $ 980 for 4 months 24 days, at 5 per cent.
9. Are $ 384 for 9 months 18 days, at 5 per cent.

Find the proceeds of—

1. A 4-months' note for $3,157, dated June 9, discounted August 25, at 4 per cent.

2. A 90-days' note for $2,250, dated August 15, discounted September 15, at 5 per cent.

3. A 3-months' note for $1,537, dated November 13, 1898, discounted December 22, 1899, at 6 per cent.

Write the note and find the proceeds when—

4. The face equals $780, the time equals 90 days, the rate of discount equals 10 per cent, the date of note equals October 14, 1900, the date of discount equals November 4, 1900.

5. The face equals $8,000, the time equals 60 days, the rate of discount equals 12 per cent, the date of note equals July 4, 1899, the day of discount equals August 1, 1899.

6. The face equals $5,000, the time equals 90 days, the rate of discount equals 9 per cent, the date of note equals June 3, 1899, the date of discount equals July 13, 1899.

7. The face equals $1,000, the time equals 45 days, the rate of discount equals 6 per cent, the date of note equals May 16, date of discount equals May 16, and the note bearing interest at 8 per cent.

Find the time when—

8. The face of a note is $500, the proceeds is $494.75, and the rate of discount is 6 per cent.

9. The face of a note is $3,000, the proceeds $2,953.50, and the rate of discount is 6 per cent.

Find the rate of discount when—

10. The face is $400, the proceeds is $398, and the time is 30 days.

11. The face is $3,450, the proceeds is $3,415.50, and the time is 60 days.

12. The face is $2,400, the proceeds is $2,372, and the time is 60 days.

13. The face is $500, the proceeds is $490, and the time is 60 days.

ACCOUNTING.

Suppose that *Brown & Power* began a partnership business, *January 1, 1899*, with a capital of $10,000. Of this amount Brown gave $2,000 in cash and $3,000 in merchandise. Power gave $2,500 in cash and four notes, as follows: $1,000, by John Smith, dated December 1, 1898, due in 60 days; $500, by Sam Black, dated January 1, 1899, due in 60 days; $600, by Richard Roe, dated November 1, 1898, due in 3 months; $400 by Wm. Jones, dated September 1, 1898, due in 90 days.

On the same day they paid "The Times," for advertising, $15, and to Thomas Kinney, repairing their storeroom, $10.50. They bought of Marshall Field, of Chicago, a bill of goods amounting to $200, of which $50 was paid in cash, and the balance was paid by a note, for $50, due in three months. They sold James Wells, on account, 3 pieces of muslin, of 45 yards each, at 11 cents a yard; 5 dozen hats at $14 a dozen.

How should a record of their business be kept?

Such a record should show—

(1) The amount of goods bought and sold.

(2) The amount of cash received and paid and amount on hand.

(3) The notes given by the firm, and given to the firm, and notes paid.

(4) The account of each individual doing business with the firm.

There are various methods of keeping accounts. The simplest system requires the use of two books, a *Day Book* and a *Ledger*.

The *Day Book* shows all transactions by which others become indebted to us, or we to others.

The *Ledger* is a collection of accounts with persons, or *personal accounts*. In the Ledger the entries of the Day Book are *posted*, or transferred to the account of the person it concerns, to show the total amount of indebtedness to us that he has incurred or discharged, and the amount of indebtedness to him incurred or discharged by us.

Other books are often used: (1) *Cash Book*, in which is recorded every receipt and expenditure of money. (2) *A Bill Book*, in which to record the amounts of promissory notes and drafts that are to be paid to, or by us, with their dates, the names of the drawers and indorsers, the times for which they are drawn, the dates when they mature, etc.

Notes and drafts to be received by us are called *Bills Receivable;* those to be paid by us are *Bills Payable.*

These accounts may be kept as accounts in the *Ledger*, with pages suitably ruled.

Every account has two sides, a *Debtor*, side, marked *Dr.*, and a *Creditor* side, marked *Cr.*

An entry on the *Dr.* side is called a *Debit;* an entry on the *Cr.* side is a *Credit.* The difference between the sum of the debits and the sum of the credits is called the *Balance.*

In the cash account every debit indicates that so much cash has been received; every credit, that so much cash has been paid. The balance shows the amount of cash on hand. Hence, *debit cash for every sum received; credit it for every sum paid.*

In the Day Book every person is made *Dr.* for whatever tends to increase his indebtedness, or diminish a debt due him. He is made *Cr.* for whatever tends to diminish his indebtedness, or increase the amount due him.

Note carefully how the above account of *Brown & Power* is recorded in the *Day Book.* Look at the Day Book record (p. 242), and notice that Brown is credited with $2,000 cash and $3,000 merchandise; a total of $5,000. *Why should he be credited with this amount?*

Power is credited with $2,500 cash and $2,500 in notes due him, a total of $5,000. *Why should he be credited for these notes?*

The *Cash* account is credited with $15 and $10.50 for advertising and repairing. Why is cash *credited?* Marshall Field is credited with $200 worth of merchandise, and is debited with $50 cash and a note of $150. His account being paid, his name is not carried to the Ledger, but the items are found under *cash* and

bills payable. James Wells is debited with muslin worth $14.85, and hats worth $70; a total of $84.85. *Why should he be debited?*

At the close of each day's business, a careful business man will transfer the record of the Day Book to the several Ledger accounts. Of the above account, the several items of cash received and expended are recorded in the cash account. (See p. 247). *Why is cash debited* with the $2,000 and $2,500? *Why is cash credited* with the $15, $10.50 and $50?

The personal account of *Brown* (page 246) gives him credit for the $5,000 he contributed to the firm.

The personal account of *Power* (see p. 246) gives him credit for the $5,000 he contributed to the firm. The notes which he turned over to the firm are recorded under *Bills Receivable.* (See p. 248.) But the note given by the firm to Marshall Field, is recorded under *Bills Payable.* (See p. 248).

The account of James Wells is recorded in the Ledger (see p. 246), and he is debited with $84.85.

Note that in the Day Book a column is given to record the page of the Ledger on which each personal account is recorded, and in the Ledger is a column to record the page of the Day Book from which the account is taken. *Why is no account opened with Marshall Field?*

The following business up to and including January 8, is posted in the Day Book. Rule writing paper for Ledger accounts, also for Cash, Bills Receivable, and Bills Payable, and transfer the Day Book record to the proper places in the other books.

January 2.—Paid a carpenter for work in the store, $37.50. Loaned S. H. Anson $500 for three days. Bought for cash, of Foss & Co., $3,150 worth of merchandise, as per invoice 2. Sold H. L. Prince, for cash, 4 dozen gents' handkerchiefs, at $3 a dozen; 1½ dozen dress patterns, at $3.50 each; 1 case, 60 pairs shoes, at $1.50 a pair.

NOTE.—No Ledger account need be opened with Prince, as the sale was made for *cash.*

January 3.—Sold G. Stone, on credit, 10 pieces ingrain carpet-

ing, 960 yards, at 75 cents; 460 yards Hess & Co.'s three-ply carpeting, at $1.10, and 120 yards Brussels carpeting, at $1.75. Sold Morse & Co., half cash, half on note at 30 days, 10 pieces linen, 40 yards each, at 30 cents; 148 yards gingham, at 15 cents; 6 dozen pair of gloves, at $11.20 per dozen. Received for petty sales, $46.80. Paid $15.60 for petty expenses.

NOTE.—The petty sales are entered on the Dr. side of the Cash Book from the footing of memoranda made throughout the day.

January 4.—The firm paid their note to Marshall Field, due April 4, ($150, less discount $3.75) $146.25. Bought merchandise, for cash, of Jas. K. Pike, $264.40, as per invoice 3. Bought merchandise on account, of G. Stone, Seattle, $463.80, as per invoice 4. Bought merchandise on account, of Samuel Bent, $247, as per invoice 5. Sold H. S. Greer, half cash, half on account, 14 reams of letter paper, at $1.80 a ream; 20,000 envelopes, at $1.50 per 1,000; 12 reams of note paper, 8 reams at $1.25 a ream, 4 reams at $1.50. Received for petty sales, $64.80.

NOTE—The firm discount their own note to Marshall Field, paying its face less the discount for the time it has to run; a memorandum to this effect is made in the column of remarks under Bills Payable, and the amount paid $146.25, is entered on the credit page of the Cash Book.

January 5.—Received of S. H. Anson, $500, amount of loan. Bought merchandise of Henderson & Co., as per invoice 6, for cash, to the amount of $347.90. Received $498.64, proceeds of Sam Black's note for $500, due January 20, discounted at First National Bank. Sold Sullivan H. Wesley, 6 dozen dictionaries, at $7.20; 2 dozen Fourth Readers, at $6.00; 4 dozen Brand's Physiologies, at $9,60; 8 writing desks, at $5.25; cash, 5% off. G. Stone paid $150, on account. Paid $75 on private account. Received for petty sales, $53.20.

NOTE 1.—When cash is paid the firm on account, a debit is required in the Cash Book, and the payer is credited with the amount. (See entries in the case of S. H. Anson and G. Stone.) When the firm pays cash on account, the person to whom it is paid is debited.

NOTE 2.—The firm having had a draft discounted, a memorandum to that effect is made opposite to the draft in the column of remarks under Bills Receivable, and the cash received is entered as a debit in the Cash Book.

January 6.—The firm paid Samuel Bent $1,500 cash, and their note (No. 2) for $975, at 60 days, in full of account. Received from James Wells, on account, $50. Received $1,000 for John Smith's note, due this day. Paid petty expenses, $20.50. Sold Roe & Son, of Ellensburg, on account, 1 case Oxford ties, 24 pairs at $1.80; 5 cases congress gaiters, 60 pairs at $3.20; 2 cases kip boots, 24 pairs at $2.10. Received for petty sales, $73.25.

NOTE.—By the payment of cash and their own note the firm discharge their indebtedness to Samuel Bent, and his account in the Ledger, may be closed. This may be done by bringing down the sum of the debits and that of the credits in the same line, to show that they balance, and drawing two parallel lines beneath.

January 7.—Sold S. H. Anson, for cash, 3 dozen silk scarfs, 1 dozen at $1.50 each, 2 dozen at $1.75; 4 dozen hats, at 90 cents a piece; 12 dozen pair hose, 9 dozen at $2, and 3 dozen at $3; 8 pieces prints, 322 yards, at 13 cents. Sold G. Stone, on account, 1 dozen sheepskin mats, at $2.50; 1 piece oilcloth, 50 yards, $1.25; half-dozen woolen rugs, at $5.50 each. Received from H. S. Greer, on account, $25. Received for petty sales, $32.10.

January 8.—Sold James Wells, on account, 12 dress patterns, at $2.25; 3 dozen ladies' handkerchiefs, at 50 cents; 6 shawls, at $12 each. Paid A. F. Power $50, on private account. Bought of Roe & Son, on account, merchandise to the amount of $2,243.75, as per invoice 7. Sold H. S. Greer, for cash, 8 reams legal cap paper, at $3.75; 6 dozen glass inkstands, at $3 per dozen; 10 reams foolscap at $2 a ream; 2 dozen grammars at 50 cents each. Received for petty sales, $39.70.

1. How many parties to a common business transaction?
2. What is the meaning of debtor? Of creditor?
3. What is the balance of an account?
4. Is an account paid by giving a promissory note for the balance?
5. In such a case, how make the Ledger account?
6. What books usually used in recording accounts?
7. Give the use of each book used.

DAY BOOK OF BROWN & POWER.

DENVER, COLO., Jan 1, 1903.

LEDGER FOLIO			ITEMS	TOTAL
	W. R. Brown,	Cr.		
	By cash capital	$2000		
	By mdse. "	3000		5000
	A. F. Power,	Cr.		
	By cash capital	$2500		
	By note from John Smith	1000		
	By note from Sam Black	500		
	By note from Richard Roe	600		
	By note from Wm. Jones	400		5000
	James Wells.	Dr.		
	To 135 yds. muslin @ 11c.		14 85	
	To 5 doz. hats @ $14.		70 00	84 85
	Cash,	Cr.		
	By advertising account		15 00	
	" repairs		10 50	25 50
	Marshall Field, cash and note			
	Mdse. as per invoice	$200		
	Cash		50 00	
	3-months' note		150 00	200 00
	JAN. 2.			
	Cash	Cr.		
	By carpenter work in store			37 50
	S. H. Anson	Dr.		
	To $500 loaned him for 3 days			500 00
	Foss & Co.	Cash		
	Mdse. bought of Foss & Co. as per invoice 2			3150 00
	H. L. Prince,	Cash		
	4 doz. handkerchiefs @ $3		12 00	
	1½ doz. dress patterns @ 3.50 each		63 00	
	1 case shoes, 60 pair @ 1.50		90 00	165 00

JAN. 3.

G. Stone, Dr.			
To 960 yds. ingrain carpet @ 75c	720 00		
To 460 yds. 3-ply " @ $1.10	506 00		
To 120 yds. brussels " @ 1.75	210 00	1436	00
Morse & Co., Cash and note			
400 yds. linen @ 30c. $120.00			
148 yds. gingham @ 15c. 22.20			
6 doz. pr. gloves @ $11.20 67.20			
Half cash	104 70		
Half 30-day note	104 70	209	40
Cash Dr.			
To receipts from petty sales		46	80
Cash Cr.			
By petty expenses		15	60

JAN. 4.

Cash Cr.			
By payment of note to Marshall Field, less discount ($150—$3.75)		146	25
By amount paid J. K. Pike for mdse. as per invoice 3		260	40
G. Stone, Cr.			
By mdse. bought on account, invoice 4		463	80
Samuel Bent, Cr.			
By mdse. on account as per invoice 5		2475	00
H. S. Greer Dr.			
To 14 reams letter paper @ $1.80	25 20		
To 12 reams note paper 8/$1.25, 4/$$.50	16 00		
To 20,000 envelopes @ $1.50 per M.	30 00	71	20
Cr.			
By Cash		35	60
Cash Dr.			
To receipts from petty sales		64	80

JAN. 5.			
S. H. Anson, Cr.			
By cash payment of $500 loaned			500 00
Cash Cr.			
By mdse. bought of Henderson & Co., as per invoice 6			347 90
Cash, Dr.			
To proceeds Sam Black's note, discounted at First National Bank			498 64
S. H. Wesley, Cash			
6 doz. dictionaries @ $7.20	43 20		
2 doz. Fourth Readers @ $6.00	12 00		
4 doz. Brand's Physiology @ $9.60	38 40		
8 writing desks @ $5.25	42 00		
	135 60		
5% off for cash	6 78		128 82
G. Stone Cr.			
By cash paid on account			150 00
W. R. Brown, Dr.			
To cash drawn on account			75 00
Cash, Dr.			
To receipts for petty sales			53 20
JAN. 6.			
Samuel Bent, Dr.			
To cash on account	1500		
To 60-day note, in full of account	975		2475 00
James Wells, Cr.			
By cash on account			50 00
Cash, Dr.			
To cash for John Smith's note			1000 00
Cash, Cr.			
By petty expenses			20 50

Roe & Son,	Dr.				
To 1 case, 24 pair, Oxford ties @ $1.80		43	20		
To 5 cases, 64 pair, cong. gaiters @ $3.20		192	00		
To 2 cases, 24 pair, kip boots @ $2.10		50	40	285	60
Cash,	Dr.				
To receipts for petty sales				73	25
JAN. 7.					
S. H. Anson,	Cash				
To 3 doz. silk Scarfs, 1/$18, 2/$21		60	00		
To 4 doz. hats @ $10.80		43	20		
To 12 doz. pair hose, 9/$2, 3/$3		27	00		
To 8 pieces prints, 322 yds., @ 13c.		41	86	172	06
G. Stone,	Dr.				
To 1 doz. sheepskin mats @ $2.50		30	00		
To 1 piece oilcloth, 50 yds., @ $1.25		62	50		
To 6 woolen rugs @ $5.50		33	00	125	50
H. S. Greer,	Cr.				
By cash on account				25	
Cash,	Dr.				
To receipts from petty sales				32	10
JAN. 8.					
James Wells,	Dr.				
To 12 dress patterns @ $2.25		27	00		
To 3 doz. ladies 'kerchiefs @ $6.00		18	00		
To 6 shawls @ $12		72	00	117	00
A. F. Power,	Dr.				
To cash paid on account				50	00
Roe & Son,	Cr.				
By mdse. as per invoice 7				2243	75
H. S. Greer,	Cash				
8 reams legal cap @ $3.75		30	00		
6 doz. glass inkstands @ $3		18	00		
10 reams foolscap paper @ $2		20	00		
2 doz. grammars @ $6		12	00	80	00
Cash,	Dr.				
To receipts from petty sales				39	70

LEDGER OF BROWN & POWER.

Dr. W. R. BROWN. Cr.

							1899 Jan.	1	By capital	1	5000	00

Dr. A. F. POWER. Cr.

							1899 Jan.	1	By capital	1	5000	00

Dr. JAMES WELLS. Cr.

1899 Jan.	3	To mdse.	1		84	85							

CASH BOOK OF BROWN & POWER.

		Cash	Dr.		Cash	Cr.
1899 Jan.	1	Rec'd of W. R. Brown as capital	$2000 00		Paid for advertising	$15 00
		Rec'd of A. F. Power as capital	2500 00		Paid for petty expenses	10 50
					Paid Marshall Field for mdse.	50 00
					By balance	$1541 02
			$7719 67			$7719 67
		To balance	$1541 02			

BILL BOOK OF BROWN & POWER.

Bills Receivable.

REC'D.	NO.	DRAWN BY	WHOSE FAVOR	DATE	TIME	DUE*	AMOUNT.	REMARKS.
Jan. 1	1	John Smith	A. F. Power	Dec. 1,'98	60 days	Jan. 30	$1000.00	Paid Jan. 6.
"	2	Sam Black	"	Jan. 1,'99	60 days	Mar. 2	500.00	Dis. Jan. 5.
"	3	Richard Roe	"	Nov. 1,'98	3 Mo.	Feb. 1	600.00	
"	4	Wm. Jones	"	Sept. 1,'98	90 days	Nov. 30	400.00	

Bills Payable.

ISSUED.	NO.	DRAWN BY	WHOSE FAVOR.	DATE.	TIME.	DUE.	AMOUNT.	REMARKS.
Jan. 1	1	Brown & Power	Marshall Field	Jan. 1	3 mo.	Apr. 1	$150.00	Dis. by selves Jan. 4.

*In some states days of grace are counted.

The names entered in a ledger are kept in the form of an alphabetical index, showing the page of the ledger on which the account of each is to be found. At the end of the month, year or at some other time, it is required to find out, whether the business is successful. Each person's account is reckoned, and a bill is made out for the "balance" due on the account. A "balance" is often made at the close of each week. Banks and large business houses balance the cash account at the close of each day.

Suppose on Jan. 8, some person wishes to buy the business of Brown & Power. If they wish to sell, they must show by some kind of *statement* the condition of the business and the amount of merchandise on hand. To do this each *Ledger* account must be *balanced* by finding the difference between the sum of the *debits* and the sum of the *credits*, and writing it on the side having the smaller sum, that the two sides will *balance*. (See form of Cash Balance.)

TRIAL BALANCE, LEDGER ACCOUNTS.

	Dr.	Cr.
W. R. Brown (prop.)....	$75 00	$5000 00
A. F. Power (prop.).....	50 00	5000 00
James Wells.....	201 85	50 00
G. Stone.....	1561 50	613 80
H. S. Greer.....	35 60	25 00
Roe & Son.....	285 60	2243 75

Will the sum of the *debits* of personal accounts taken in their order in the Day Book equal the sum of the *debits* as taken from the Ledger? Why? Is the same true of the *credits?*

Drawing up a statement. The values of the unsold merchandise is ascertained to be $8,250, by a careful inventory of the several items on hand. A *statement* of *resources* and *liabilities* may now be prepared, from the Ledger balances of personal accounts, (Trial Balance) the cash balance as shown in the Cash Book, and the Bills Receivable and Bills Payable accounts.

An excess of debits in a personal account is a resource; an excess of credits is a liability. The merchandise on hand and real estate belonging to the firm are resources. If the books have been properly kept, they will show the resources and liabilities of the business as follows:

STATEMENT.

RESOURCES.		LIABILITIES.	
Mdse. on hand....	$8250.00	Roe & Son.....	$1958.15
Real estate.....	1000.00	Bills payable.....	975.00
James Wells.....	151.85	Balance	$10,072.72
G. Stone.....	947.70		
H. S. Greer.....	10.60		
Cash balance.....	1541.02		
Bills receivable...	1104.70		
	$13,005.87		$13,005.87

Present worth, (excess of resources) $10,072.72.

Finding the Profit or Loss is now easy, for it is the difference between the present worth and the net capital—*profit*, if the present worth is greater; if not, *loss*.

The original capital was $10,000; but each partner having drawn out some cash, the net capital is less than the original capital by the amounts thus withdrawn.

Present worth, as above		$10,072.72
Original capital..		10,000.00
Profit.....		$72.72
Cash drawn out by Brown.....	$75.00	
Cash drawn out by Power.....	50.00	125.00
Total net profit.....		$197.72

How will Brown and Power divide the $72.72?

The above system of bookkeeping is called *Single Entry*. It is adapted to any small business, and may be modified to suit conditions.

Every person should keep a personal expense account, debiting it with such sums as are from time to time earned, and crediting it with what is paid out. By balancing the account at the close of each month, it may be found whether the balance in the book agrees with the balance of cash on hand.

A farmer should keep an account with his farm, or with any particular branch of his business. For instance, he may open an account with his "Farm," *debiting* it with whatever is expended on it, with the value of labor estimated at current rates, and with the interest on the value of the land; and *crediting* it with the value of what the field produces at the regular market price. He can tell by the balance, at the close of the year, what he has gained or lost.

A more elaborate system, known as *Double Entry*, is required for wholesale, or other extensive business.

MERCANTILE FORMS AND DEFINITIONS.

COMMERCIAL PAPER includes notes, checks, drafts, orders, and bills of exchange.

A NOTE is a written promise by one person to pay to another a specified sum at a certain time.

The person who promises is called the *maker*, and the person to whom he promises is called the *payee*.

The *holder* of a note is its legal owner.

The *face* of a note is the sum of money promised.

A NEGOTIABLE NOTE is one which is made payable to the *bearer*, or to the *order* of the *payee*. A negotiable note can be sold or transferred.

A note is *non-negotiable* when it is payable only to the person or persons named in the note.

A JOINT NOTE is one signed by two or more persons, who are jointly liable for its payment.

A JOINT AND SEVERAL NOTE is a note signed by two or more parties, who are jointly and singly liable for its payment.

A DEMAND NOTE is one payable on demand.

DEMAND NOTES are payable on presentation without grace, and bear legal interest after a demand has been made. An indorser on a demand note is responsible only for a limited time, variable in different States.

Notes falling due on Sunday, or on a holiday, must be paid the day previous.

Notes made on Sunday are void.

All notes should contain the words "value received."

$400. TUCSON, *Sept. 11*, 1899.

Sixty days after date, I promise to pay *John Jones*, or order, *Four hundred* $\frac{00}{100}$ Dollars, value received. *T. W. Sterling.*

Form of Promissory Note.

DEMAND NOTE, WITH INTEREST.

$200. DENVER, Sept. 18, 1899.

On demand, I promise to pay Dr. J. C. Roberts, or order, two hundred dollars, with interest, value received.

W. R. BROWN.

JOINT AND SEVERAL NOTE.

$700. OMAHA, Sept. 10, 1899.

Three months after date, we jointly and severally promise to pay J. J. Hill, or order, seven hundred dollars, with interest at 6%, at the Capital National Bank. Value received.

W. R. BROWN.
A. F. POWER.

SIGHT DRAFT.

$100. PORTLAND, Oct. 1, 1899.

At sight, pay to the order of David Thomas one hundred dollars, value received, and charge to account of

W. A. DALTON.

To MR. J. R. MASON, Seattle.

$400. BERKELEY, *Oct. 8*, 1899.

Sixty days after date pay to *Butler Bros.*, or order, *Four hundred* $\frac{00}{100}$ Dollars, value received, and charge to account of *R. R. Perkins.*

To *First National Bank, Spokane.*

Form of Time Draft.

CHECK.

No. 360. EL PASO, December 22, 1900.

$1200.

OLYMPIA STATE BANK.

Pay to the order of W. H. Andrews twelve hundred dollars.

JAS. L. HARMON.

RECEIPT.

$50. DAYTON, September 28, 1899.

Received of O. V. Linn fifty dollars, on account (or in full of all demands). CUSHMAN & PLUMMER.

ORDER FOR MERCHANDISE.

OLYMPIA, December 20, 1901.

Messrs. McWilliams & Coulter:

Please send us immediately 10 barrels flour, Capital City brand, and charge to our account. J. BETHEL & Co.

LETTER OF CREDIT.

SEATTLE, October 3, 1901.

Gentlemen:

Please let Mr. Daniel Eddy, the bearer of this letter, have goods to any amount he may desire not exceeding five hundred dollars, ($500) and hold us responsible for payment in case of any default on his part. Should Mr. Eddy fail to meet his obligations at any time, give us immediate notice.

Yours truly,

F. A. DANAHOWER & Co.

EXCHANGE.

If A lives in St. Louis, and owes B in Chicago for merchandise $500, and C in Chicago owes D in St. Louis $500 for fruit, they can both pay their debts by sending $500 each way. But it will save expense and time if A in St. Louis can pay D, and C in Chicago can pay B. This may be done by means of a money order through the postoffice, but more frequently it is done by means of *checks* and *drafts* on the banks.

Because one debt is thus exchanged for another this method is called *Exchange.* The banks charge for doing the business a small sum, not exceeding the actual cost of sending the money, which charge is called the *rate of exchange.*

The rate of exchange between two places depends on their relative trade. If the trade between St. Louis and Chicago is equal, exchange is at par. If St. Louis owes Chicago, the call in St. Louis for drafts on Chicago exceeds the demand in Chicago for drafts on St. Louis; and, as a consequence, the drafts in St. Louis on Chicago are at a premium, while in the latter city St. Louis drafts are at a discount.

A DRAFT is a written order by one person on another for the payment of a specified sum. Drafts on foreign countries are called *bills of exchange.*

A BANK DRAFT is a written order by one bank on another for the payment of a specified sum to a person named therein, or to his order. Drafts are made payable at sight, on demand, or at a certain time after date, or after sight.

A BANK CHECK is a draft on a bank. Checks are not considered as actual payment until they are paid, but in the course of business are regarded as cash.

SIGHT DRAFTS, like checks, are presented for payment.

Drafts drawn on time are first presented for *acceptance.*

The person to whom the draft is presented *accepts* it by writing across the face the word, "*Accepted,*" with the date, over his signature.

1. What is the cost of a \$2,000 draft, at a $\frac{3}{4}\%$ premium?

SOLUTION.

\$2000 × 0.00$\frac{3}{4}$ = \$ 15.00, premium,
\$2000 + \$15.00 = \$2015.00, the cost.

2.

\$300. SAN FRANCISCO, May 8, 1903.

Sixty days after sight, pay to H. Wagner, or bearer, three hundred dollars, value received, and charge the same to the account of WHITAKER, RAY & Co.

To H. Wilson, Seattle.

What is the cost of this draft, at $\frac{1}{2}\%$ discount?

SOLUTION.

\$320 × 0.005 = \$ 1.50, discount,
\$320 — \$1.50 = \$318.50, the cost.

3. What is the cost of a draft for \$1,000, payable in 60 days after sight, at 5% interest, exchange at $\frac{1}{2}\%$ premium?

SOLUTION.

Premium on \$1 = \$.005,
Discount on \$1 for 63 days = \$.00875,
The cost of exchange for \$1 = \$1.005 — 1.00875 = \$.99625,
The cost of \$1000 = 1000 × \$.99625 = \$996.25.

Find the cost of—

4. A sight draft for \$500, $1\frac{1}{4}\%$ premium.

5. A sight draft for \$300, $1\frac{1}{4}\%$ discount.

6. A 60-day draft for \$4,000, $\frac{2}{5}\%$ premium, interest 6%.

7. A 60-day draft for \$4,000, $\frac{2}{5}\%$ discount, interest 6%.

FOREIGN EXCHANGE.

A *Bill of Exchange* is an open letter of request from one person to another to pay to a third party named therein, a certain sum of money, at a specified time and place. Three copies are made, and are sent by different ways to the drawee, so that in case one is lost, another may reach him. There are three parties to a bill of exchange, and possibly four, viz:

1. He who writes the bill, called the *drawer* or *maker*.
2. He to whom it is directed, called the *drawee*.
3. He to whom the money is ordered to be paid, called the *payee;* and
4. He who purchases a bill of exchange, called the *buyer* or *remitter*.

£6000. SAN FRANCISCO, July 30, 1900.

Sixty days after sight of this my first Bill of Exchange (second and third of the same date and tenor unpaid)* pay to John Smith, or order, six thousand pounds sterling, with or without further notice.

BROWN & POWER.

To Williams Bros., London.

The calculation of foreign exchange is the same as that for domestic exchange, the only new element being the rate of exchange in expressing values of one country in the money of another.

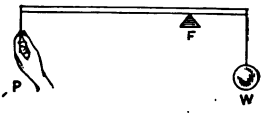
*The Second Bill of the Set would read, "of this Second of Exchange (First and Third of the same date and tenor unpaid)." The Third would run, "of this Third of Exchange (First and Second, etc.)"

1. Define a *promissory note.*
2. Define *maker; payer; holder; face.*
3. Define a *negotiable* note; a *non-negotiable* note.
4. Define a *demand* note; a *joint* and *several* note.
5. Give the reason for writing "value received" in a note.
6. Explain the operation of *exchange,* in facilitating business between cities.
7. Explain how the state of trade regulates the *rate of exchange.*
8. Explain the difference between a check and a draft.
9. Explain the *presentation* and *acceptance* of a *time draft.*
10. Is a check considered full payment of a debt? Why is a check more convenient than the real money?
11. When is a check worthless by fault of the drawer? Of the bank on which it is drawn?

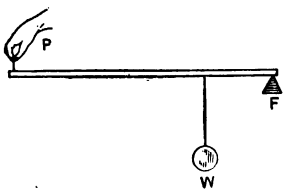
LEVERS.

THE LEVER is a bar which moves around a centre of motion called a *fulcrum*. There are three classes of levers—

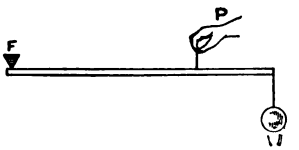
First. When the fulcrum is between the weight and the power.



Second. When the weight is between the power and the fulcrum.



Third. Wher the power is between the fulcrum and the weight.



The power and weight will balance when the weight multiplied by its distance from the fulcrum, is equal to the power multiplied by its distance from the fulcrum. This does not take into consideration the weight of the lever itself, which slightly modifies the ratio between the power and the weight.

1. In a lever of the first class, the fulcrum is at the middle point. What power will balance a weight of 100 pounds?

2. In a lever of the second class, the weight is at the middle point. What power will sustain a weight of 100 pounds?

3. In a lever of the third class, the power is at the middle point. What power will sustain a weight of 100 pounds?

4. A lever of the first class is 16 feet long, and a weight of 100 pounds is 4 feet from the fulcrum. What power will balance it?

5. A lever of the first class is 12 feet long, and a weight of 100 pounds is 2 feet from the fulcrum. What power will balance it?

1. $\frac{9}{10} - \frac{3}{4}$.

2. $\frac{7}{12} - \frac{4}{18}$.

3. $4\frac{2}{3} + 3\frac{3}{4}$.

4. $5\frac{1}{2} + 2\frac{5}{6}$.

5. $\frac{8\frac{2}{5}}{10\frac{1}{2}} + \frac{5\frac{1}{3}}{6\frac{2}{5}}$.

6. $\frac{5\frac{1}{4}}{9\frac{1}{3}} \quad \frac{1\frac{5}{6}}{7\frac{6}{7}}$.

7. $\frac{9}{10} - \frac{8}{9}$.

8. $\frac{7.2}{8} - \frac{.6}{.72}$.

9. $\frac{.25}{.075} + \frac{.075}{.25}$.

10. $\frac{.2}{.03} - \frac{.02}{.3}$.

11. $\frac{4.5}{9} + \frac{3.75}{7.5}$.

12. $\frac{2\frac{2}{3}}{4.5} + \frac{1.3}{2\frac{3}{5}}$.

Find the missing term of each of the following:

13. 6 : 8 :: 12 : —.

14. 9 : 25 :: — : 15.

15. 2 : — :: 7 : 5.

16. $\left\{\begin{matrix} 3 : 8 \\ 7 : 5 \end{matrix}\right\}$ :: 56 : —.

17. $\left\{\begin{matrix} 2\frac{2}{3} : 7\frac{1}{2} \\ 4\frac{1}{2} : 6\frac{2}{5} \end{matrix}\right\}$ :: 12 : —.

18. $4\frac{1}{2} : 5\frac{2}{5} :: 6\frac{1}{4} : —$.

19. .06 : 2.5 : — : 15.

20. .012 : $.01\frac{5}{7}$ :: $.5\frac{3}{5}$: —.

21. $\left\{\begin{matrix} .03 : 1.2 \\ 1.5 : 8 \end{matrix}\right\}$ :: —: .096.

22. $\left\{\begin{matrix} 80 : .08 \\ 300 : .003 \end{matrix}\right\}$ :: 100000 : —.

How much shall I gain or lose by selling—

23. Articles costing $ 9 at an advance of 12 per cent?

24. Articles costing $ 27.50 at a loss of 16 per cent?

25. Articles costing $ 12.50 at an advance of $12\frac{1}{2}$ per cent?

26. Articles costing $ 25 at a loss of 25 per cent?

27. Articles costing $ 8.70 for 112 per cent of cost?

28. Articles costing $125 for 125 per cent of cost?

29. Articles costing $ 96 for 96 per cent of cost?

30. Articles costing $ 8 for $87\frac{1}{2}$ per cent of cost?

For how much must I sell—

31. Articles costing $ 4 to gain $16\frac{2}{3}$ per cent?

32. Articles costing $ 7.50 to gain 15 per cent?

33. Articles costing $12 to lose $12\frac{2}{3}$ per cent?

34. Articles costing $ 3.56 to lose 25 per cent?

35. Articles costing $ 3.56 to gain 25 per cent.

State the gain or loss per cent—

1. If the cost is \$ 6.00 and the selling price is \$ 7.00.
2. If the cost is \$ 5.00 and the selling price is \$ 6.50.
3. If the cost is \$ .50 and the selling price is \$ .62½.
4. If the cost is \$ .62½ and the selling price is \$ .50.
5. If the cost is \$172.25 and the selling price is \$137.80.
6. If the cost is \$137.80 and the selling price is \$172.25.
7. If the cost is \$ 15.00 and the selling price is \$ 15.75.
8. If the cost is \$ 15.75 and the selling price is \$ 15.00.

What per cent of the selling price—

9. Is a loss of 10% of the cost?
10. Is a gain of 10% of the cost?
11. Is a loss of 20% of the cost?
12. Is a gain of 20% of the cost?
13. Is a gain of 50% of the cost?
14. Is a loss of 50% of the cost?
15. Is a gain of 100% of the cost?
16. What per cent is lost if the loss equals 10 per cent of the selling price?

SOLUTION.

The loss added to the selling price equals the cost. Hence, if I lose 10 per cent of the selling price, the cost must be 110 per cent of the selling price, and the loss must be $\frac{10}{110} = \frac{1}{11} = 9\frac{1}{11}$ per cent of the cost.

What per cent is gained or lost—

17. If the loss equals 25 per cent of the selling price?
18. If the loss equals 50 per cent of the selling price?
19. If the loss equals 20 per cent of the selling price?
20. If the gain equals 25 per cent of the selling price?
21. If the gain equals 16⅔ per cent of the selling price?
22. If the gain equals 6¼ per cent of the selling price?
23. If the gain equals 20 per cent of the selling price?
24. If the gain equals 33⅓ per cent of the selling price?
25. If the gain equals 12½ per cent of the selling price?

What is the cost of goods—

1. On which the gain is \$2 by gaining 10 per cent?
2. On which the gain is \$ .04 by gaining 50 per cent?
3. On which the loss is \$ .45 by losing 15 per cent?
4. On which the loss is \$5.20 by losing $14\frac{2}{7}$ per cent?

What is the selling price of goods—

5. On which the gain is \$ 6 by gaining 6 per cent?
6. On which the loss is \$ 6 by losing 6 per cent?
7. On which the gain is \$ .09 by gaining 3 per cent?
8. On which the loss is \$ .09 by losing 3 per cent?
9. On which the gain is \$15.50 by gaining $16\frac{2}{3}$ per cent?
10. On which the loss is \$15.50 by losing $16\frac{2}{3}$ per cent?
11. What is the cost of goods on which the gain is 12 per cent by selling for \$3.08?

SOLUTION.

Since the gain is 12 per cent, the selling price must equal 112 per cent of the cost. Hence, 100 per cent of the cost, or the cost itself, must equal $\frac{100}{112}$ of the selling price. $\frac{100}{112}$ of \$3.08 = \$2.75 = cost..

12. What is the cost of goods on which I lose 12 per cent by selling for \$3.08?

SOLUTION.

Since I lose 12 per cent, my selling price must equal 88 per cent of the cost. Hence, 100 per cent of the cost, or the cost itself, must equal $\frac{100}{88}$ of the selling price. $\frac{100}{88}$ of \$3.08 = \$3.50 = cost.

What is the cost of goods on which I—

13. Gain 20 per cent by selling for \$1,728?
14. Lose 20 per cent by selling for \$1,728?
15. Gain $33\frac{1}{3}$ per cent by selling for \$15.50?
16. Lose 40 per cent by selling for \$40?
17. I bought a carriage for \$125, and, after paying \$12.50 for having it painted and varnished, I sold it for \$150. What was my gain per cent?

1. If the minuend be 600 and the difference between the remainder and subtrahend 100, what are the last two terms?

2. Two men having met on a journey, found that they had both together traveled 1,200 miles; but one had traveled 200 miles more than the other. What was the distance each traveled?

3. Suppose four boys together weigh 435 pounds, and that it should so happen that three of them have the same weight; but the other weighs 15 pounds more. What would be the weight of each boy?

4. Divide $500 so that A may have $50 more than B, and C $100 more than B.

5. My purse and money are worth $100; but the money is worth 19 times more than the purse. How much money in the purse?

6. If a clerk, whose salary for 4 years amounted to $2,000 received $50 advance for each successive year after the first, what was his salary for each year?

7. If I buy a lot of oranges at 3 cents each, and as many more at 5 cents each, and sell them at 4 cents each, do I gain or lose by the operation?

8. If I buy a number of oranges at 3 for 1 cent, and as many more at 5 for 1 cent, and sell them at 4 for 1 cent. Do I gain or lose by the operation?

9. If I expend a sum of money for oranges at 3 cents each, and another equal sum for another lot at 5 cents each, how much do I gain on each cent expended, if I sell them at 4 cents each?

10. If A can do a piece of work in 3 days, B in 4 days, and C in 5 days, how many times longer will it take B to do it alone than it will take A and C together to do it?

11. A man bought apples at 5 cents a dozen, half of which he exchanged for pears at the rate of 8 apples for 5 pears. He then sold all his apples and pears at a cent each, and thus gained 19 cents. How many apples did he buy, and how much did they cost?

1. C resides 3 times as far from the city as A, and B 5 times as far as C, and to meet in the city they must all travel 380 miles. What distance from the city does each reside?

2. A man bought a horse, saddle, and bridle for $318, and paid 20 times as much for the horse as for the saddle, and $\frac{1}{5}$ as much for the bridle as for the saddle. What did the bridle and saddle both cost?

3. Fifteen years ago I was three times as old as my eldest son, who was then but 15; but am now only twice as old. What are our present ages?

4. A company at a tavern spent $\$3\frac{6}{10}$, and each of them had as many dimes to pay as there were persons in the company. How many persons were there?

5. Suppose you sell 4 bushels of oats at $37\frac{1}{2}$ cents a bushel, and pay $62\frac{1}{2}$ cents for 5 pounds of cheese, and lay out the balance in pins at $6\frac{1}{4}$ cents a paper. How many papers of pins will you have, and how much will the cheese cost by the pound?

6. If 1 pound of tea be worth $2\frac{1}{2}$ pounds of coffee, and 1 pound of coffee be worth $3\frac{1}{2}$ pounds of sugar, what will be the value of 112 pounds of tea, sugar being worth 10 cents a pound?

7. If 3 pounds of tea be worth 4 pounds of coffee, and 6 pounds of coffee be worth 20 pounds of sugar, how many pounds of sugar may be had for 18 pounds of tea?

8. A bank charged $2.52 for discounting a note at 60 days. What was the amount of the note, the rate of discount being 6%?

9. A fox is 52 rods before a dog. The fox runs 20 rods and the dog 25 rods in a minute. How many rods must the dog run to catch the fox?

10. From a vessel containing 50 gallons of wine 10 gallons were drawn off and the vessel filled with water. How many gallons of the mixture must be drawn off that 32 gallons of pure wine will remain?

11. A and B have the same income. A saves $\frac{1}{5}$ of his, but B, by spending $120 per annum more than A, at the end of 10 years finds himself $200 in debt. What was the income of each?

1. What will 2 barrels of pork cost at 12½ cents per pound?

2. If you purchase 29 yards of ribbon for 6¼ cents per yard, and give the shopkeeper a five-dollar bill, how much change should he give you?

3. A cistern has three spouts, and one will fill it in two hours. another in 3 hours, and the third in 4 hours. In how many hours would it be filled by them all together?

4. A cistern is filled by two spouts, in 4 and 12 minutes, respectively, and is emptied by a tap in 16 minutes. How many minutes will have elapsed before it is filled, when they are all left open and running?

5. The aggregate of $\frac{2}{3}$ and $\frac{3}{5}$ of a sum of money is \$60. What is the sum?

6. Find a fraction which multiplied by $\frac{2}{3}$ of $\frac{3}{4}$ of 3, will make a product of $\frac{2}{5}$?

7. Find what number of times 2s. 6d. is contained in 1£.

8. Five % of 120 is 8 less than 5% of what number?

9. A and B at opposite points of a field 135 rods in compass, start to go around it in the same course at the same time, A, at the rate of 11 rods in 2 minutes, and B, 17 rods in 3 minutes. How many times around the field will each go before the one will overtake the other?

10. A and B together can build a wall in 8 days, A and C can together build it in 10 days, and B and C can together build it in 12 days. What time required for all together to build it?

11. An agent spent \$3,000 for horses, cows, and sheep. The horses cost \$120, the cows \$3, and the sheep \$5 each. There were six times as many sheep as cows, and one third as many horses as sheep. Find the number of each.

12. A, B, and C do a piece of work in a certain time. A and B together do $\frac{4}{5}$ of it, and B and C together do $\frac{2}{3}$ of it. What part does B do alone?

13. A man owns the southeast quarter, and the northeast quarter of the southwest quarter, of a section of land. What will it cost to fence his land in one field at 80 cents per rod?

1. What per cent of a long ton is a common ton?

2. What per cent of a pound avoirdupois is a pound troy?

3. What per cent of an ounce avoirdupois is an ounce troy?

4. A person having $\frac{2}{5}$ of a coal mine sells $\frac{3}{4}$ of his share for $600. What is the whole mine worth?

5. A can do a piece of work in 5 days, B in 3 days, and C in 10 days. How long would they jointly require to do it?

6. If $\frac{1}{2}$ the trees in an orchard bear apples, $\frac{1}{4}$ pears, and $\frac{1}{6}$ plums, 40 of them peaches, and 10 cherries, what number of trees does the orchard contain?

7. A person buys a state warrant for 87½%. If it is paid in 2 years, what rate of interest does he receive on his investment, the warrant drawing 7% interest?

8. How change pounds troy to an equivalent in pounds avoirdupois?

9. A vessel holds 41.2 Kl. of milk. What is the milk worth at 7 cents a liter?

10. Find the value of $\frac{(\frac{1}{12}+)\frac{1}{3}\times(\frac{5}{6}-\frac{3}{4})}{\frac{1}{8}\div\frac{1}{2}}$

11. How deep must I make my bin, which is 21 feet long and 6 feet wide, in order that it may hold, when even full, 800 bushels of wheat?

12. An attorney collected a debt, retained his commission of 5%, and remitted his client $376.20. How much did he collect?

13. What distance does a workman save in a year by going diagonally across a lot 400 feet wide and 600 feet long, instead of going along the sides at right angles, if he travels it 4 times a day for 313 days?

14. A room is 33⅓ feet long, 20 feet wide, and 15 feet high. What is the distance from the lower southeast corner to the upper northwest corner?

15. January 13, 1897, the opening of S. Pac. railroad stock was 13½, and the closing was 12. If a speculator bought 1,000 shares at opening and sold at closing, paying ⅛% brokerage each way, how much was his loss?

1. What is that number of which $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$ is 45?

2. When $5\frac{1}{2}$ bushels of wheat cost $\$2\frac{3}{4}$, what is the price per bushel?

3. What number is that to which if its half be added the sum will be 12?

4. What number is that to which if its half, its third, and its fourth be added the sum will be 25?

5. The double and the seventh of a certain number will make 45. What is that number?

6. A horse will eat 8 hundredweight of hay in a month, a cow 4 hundredweight, and a calf 3 hundredweight. In what time will they all consume a ton of hay?

7. A person being asked how much money he had, replied that if $\frac{1}{2}$ and $\frac{1}{4}$ of it, with \$40 more be added to it, the sum would be 4 times as much as he had. What sum had he?

8. What number is that which, if increased by its half and its third, and 13 more, will be doubled?

9. Express $\frac{5}{8}$ of an acre in rods.

10. What is the interest on \$784.25 from August 7, 1894, to July 19, 1897, at 8%?

11. A certain hall is 24 feet long, 18 feet wide, and $22\frac{1}{2}$ feet high. The distance is required from one of the lower corners to an opposite upper corner.

12. Three boys travel around a circular bicycle track, the first going around in $\frac{1}{2}$ an hour, the second going around in $\frac{2}{3}$ of an hour, and the third going around it in $\frac{3}{8}$ of an hour. How long must they travel that they may meet at the starting point, and how many times will each have gone around the track?

NOTE.—How many times will each go around in one hour? Can you clear of fractions?

13. For what must I sell goods which were sold to me at 30%, 10%, 5% off a list price of \$830, to gain 20%?

14. What per cent is made by a druggist who buys medicine at \$5 a pound avoirdupois weight, and sells it at 48 cents an ounce apothecaries' weight?

1. The National Base Ball League closed the season of 1899 with the following standings. Fill in the blanks.

NAMES OF CLUBS	Games won	Games lost	Per cent won
Brooklyn	102		68.45%
Boston	95	57	
Philadelphia	94		61.84%
Baltimore		62	58.39%
St. Louis	84	67	
Cincinnati	83		54.99%
Pittsburg		74	50.96%
Chicago	75		51.02%
Louisville	76	77	
New York		92	39.87%
Washington	55	96	
Cleveland		133	13.63%

2. Which is the more profitable investment, Bank of California stock paying 12% dividends, selling at $240 per share, or First National paying 9% and selling for $160 per share?

3. A note for $800, dated March 1, 1898, bearing interest at the rate of 10%, is indorsed as follows: August 10, 1898, $200; September 1, 1898, $50; January 1, 1899, $15. What was due March 1, 1899?

4. What is the bank discount of a note for $300, dated December 11, 1896, bearing interest at $\frac{3}{4}$% per month, payable 6 months from date, discounted March 1, 1897, at $1\frac{1}{2}$% per month?

5. A, B, and C entered into business as partners, A and B each putting in $5,000, and C a certain sum. At the end of two years A took out $1,000, B $2,000, and C $3,000, and at the end of the fourth year their loss was $3,600, of which C's part was $1,050. Find A and B's share of the loss, and C's original capital.

1. Mr. Jones bought the southeast quarter of the northwest quarter of section 10, and also the east half of the southwest quarter of the same section. What is the length of fence that will enclose both pieces?

2. A man had a field 54 rods long, and 24 rods wide, which he traded for a square field of the same area. How many more rods of fence required to enclose the former field than the latter?

3. If 4 masons build 27 yards of wall in 5 days, working 9 hours a day, in how many days will 32 masons build 81 yards of a similar wall if they work 10 hours a day?

4. A ladder 50 feet long touches the side of a building at a point 30 feet above the ground. How far is the bottom of the ladder from the base of the house?

5. Find the difference between the true and the bank discount on a non-interest bearing note for $6,285, for 9 months and 15 days, money being worth 6%.

6. A hall is 24 feet long and 15 feet wide, and the area of the walls and ceiling is 1,296 square feet. What is the height of the walls?

7. A merchant makes a reduction of $8\frac{1}{3}\%$ in the selling price of an article. This reduction will leave him still a profit of $6\frac{1}{4}\%$. The article had cost him $176. What is the reduced price? What was the old price before the reduction?

8. A, B, and C formed a partnership to operate a dairy. A contributed $500, B $700, and C 24 cows. The first year they gained $1,728, of which C received $768. What were A's and B's gain and the average value of C's cows?

9. Five men can do a piece of work in 9 days. How soon after beginning must they be joined by two more so as to complete the work in 8 days?

Note.—5 men, in 8 days, can do $\frac{5}{9}$ of the work. $\frac{1}{9}$ of the work has been done by the two additional men. One man can do $\frac{1}{45}$ in one day. Two men can do $\frac{2}{45}$ in one day. How long must they work to do $\frac{1}{9}$?

10. A room in the form of a cube contains 3,375 cubic feet. At $1.75 per sq. yd., how much will it cost to carpet the floor?

1. Divide \$299 into three parts which shall be to each other as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{5}$.

2. Three contractors do a piece of work for \$7,832. The first man furnishes 25 men for 24 days, 9 hours a day; the second, 20 men for 25 days, 10 hours a day, and the third, 30 men for 20 days, 12 hours a day. What does each of the contractors receive?

3. How long will it require one of the heavenly bodies to move through a quadrant, if it moves at the rate of 3′ 12″ per minute?

4. How many feet of inch lumber in a box 6 ft. 6 in. long, 4 ft. 2 in. wide, and 3 ft. 2 in. deep, inside measurement?

5. A milkman mixes a gill of water with every pint of milk. How many gallons will he thus make out of 48 quarts of pure milk?

6. A tree broken off 21 feet above the ground, and resting on the stump, touches the level ground 28 feet from the base of the stump. What was the height of the tree?

7. The minute hand of a town clock is 16 inches long. How far will the outer end move in 12 minutes?

8. A cylindrical tank is 10 feet in diameter and 30 feet long. How many gallons in it, if filled to two-thirds its capacity?

9. How much 2-inch plank required for a 5-foot walk on the street sides of a corner lot 4×8 rods, the walk to be placed 2 feet 6 inches from the fence? How much will it cost at \$16 per M.?

10. A merchant marks certain goods so as to make 50%. If he sells to make $12\frac{1}{2}$%, what discount has he allowed?

11. January 1, 1899, A and B commenced business with \$2,400 capital, each furnishing \$1,200; March 1, B put in \$1,200 more, and April 1 they took in C, with \$3,000. At the end of a year how should a gain of \$2,000 be apportioned?

12. A takes 6 steps while B takes 7, but 4 of A's steps equal 5 of B's. Which is the faster walker?

13. If 16 men have enough provisions to last them 20 days, how long will 3 times the quantity last 72 men?

1. A man started from Chicago and traveled until his watch was 1 hour 41 minutes 40 seconds too fast. Which way and how far did he go?

2. A man pays \$750 a year rent for a house worth \$10,800. What will he gain or lose in 4 years, if he borrows money at 6% to purchase the house?

3. Reduce to simplest terms the fraction $\frac{4\frac{3}{4}+\frac{2}{3}}{3\frac{1}{7}-1\frac{3}{4}}\times 2$.

4. A boy is now 15 years old. How much must be invested for him at 7% simple interest, that when he is 21 he may have \$35,500?

5. Given the proportion, $a : b :: c : d$, to derive the equations: $ad = bc$, $a = \frac{bc}{d}$, $b = \frac{ad}{c}$, $c = \frac{ad}{b}$, $d = \frac{bc}{a}$. Prove these formulas by solving problems on page 104.

6. What three general principles are derived from these formulas?

7. In operations of percentage, the *base* times the *rate* is the *percentage*. Let the initial letters represent these elements of a problem, and derive the following equations: $r = \frac{p}{b}$, $b = \frac{p}{r}$. Prove these formulas by solving problems on pages 130 and 131.

8. In interest problems, the *interest* is the product of the *principal*, *rate*, and *time*. Let the initial letters represent these elements, and derive the following equations: $t = \frac{i}{pr}$, $p = \frac{i}{tr}$, $r = \frac{i}{pt}$. Prove these formulas by solving problems on pages 153, 155, 156.

9. Since the *amount*, in interest problems, is the sum of the principal and interest, show how to derive the following equations: $p = \frac{a}{1+rt}$, $r = \frac{a-p}{pt}$, $t = \frac{a-p}{pr}$. Prove these formulas by original problems.

ARITHMETICAL PROGRESSION.

A *series* of numbers, which succeed each other by a common difference, is said to be in *Arithmetical Progression.* When the terms are constantly increasing, the series is an *ascending progression;* when constantly decreasing, the series is a *descending progression.* Thus, 1, 3, 5, 7, 9, etc., is an *ascending progression;* and 10, 8, 6, 4, 2, is a *descending progression.*

The terms of an arithmetical progression may be fractional; as—

$\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, etc. Common difference is $\frac{1}{2}$.

$\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, etc. Common difference is $\frac{1}{3}$.

In algebra, when the common difference is positive, the series is called an *ascending* series, or an *ascending* progression; when the common difference is negative, a *descending* series. Thus—

$$a,\ a+d,\ a+2d,\ a+3d, \text{ etc.},$$

is an ascending series, in which the common difference is d; and

$$a,\ a-d,\ a-2d,\ a-3d, \text{ etc.},$$

is a descending series, in which the common difference is $-d$.

In arithmetical progression, there are five things to be considered—

(1) *The first term.* (2) *The last term.* (3) *The common difference.* (4) *The number of terms.* (5) *The sum of all the terms.*

These quantities are so related to each other, that any three of them being given, the remaining two can be found.

For convenience in statement—

a = the first term, l = the last term, d = the common difference, n = the number of terms, S = the sum of all the terms.

In the arithmetical series 2, 5, 8, 11, and 14, each term may be expressed as follows—

1st term	$= 2$		$=$ 1st term.
1st term + common difference	$= 2+$	$3 =$	$5 =$ 2d term.
1st term $+2\times$ common difference	$= 2+$	$6 =$	$8 =$ 3d term.
1st term $+3\times$ common difference	$= 2+$	$9 =$	$11 =$ 4th term.
1st term $+4\times$ common	$= 2+$	$12 =$	$14 =$ 5th term.

Thus it is seen that, in an ascending series, the second term is found by adding the common difference once to the first term; the third term, by adding the common difference twice to the first term, etc. A similar explanation may be given when the series is descending. Hence, to find the last term:

Multiply the common difference by the number of terms less one, and add the product to the first term, if the series be ascending; and subtract it if the series is descending.

Or, expressed by algebraic formula,

$$l = a+(n-1)\,d, \text{ ascending series.}$$
$$l = a-(n-1)\,d, \text{ descending series.}$$

1. Find the last term of a series of which $a = 3$, $d = 2$, and $n = 7$.

2. Given $a = 29$, $d = -2$, $n = 14$, to find l.

3. Given $a = 1$, $d = \frac{1}{2}$, $n = 17$, to find l.

4. Given $a = 20$, $d = 10$, $n = 100$, to find l.

Other formulas may be derived from the above, as follows:

$$(1)\quad l = a + (n-1)\,d,$$

Transpose a $=$ $(2)\quad l - a = (n-1)\,d,$

Divide (2) by $(n-1)$ $=$ $(3)\quad \frac{l-a}{n-1} = d,$

Divide (2) by d $=$ $(4)\quad \frac{l-a}{d} = n-1,$

Transpose 1 $=$ $(5)\quad \frac{l-a}{d}+1 = n.$

In (1) transpose $(n-1)\,d$ $=$ $(6)\quad l-(n-1)\,d = a.$

5. Given $a = 5$, $l = 47$, $n = 7$, to find d.

6. Given $a = 3$, $l = 300$, $n = 10$, to find d.

7. Find one arithmetical mean between 7 and 17.

8. Find three arithmetical means between 3 and 27.

9. Find five arithmetical means between 7 and 31.

10. A merchant deposits in the bank at the close of the day \$50, and increases the amount each day by an equal amount, so that the amount deposited the twelfth day was \$160. State each day's deposit.

Write the series of 2, 5, 8, 11, 14, as follows:

$2+5+8+11+14$ = sum of the series.

Reverse order, $14+11+8+5+2$ = sum of the series.

$16+16+16+16+16$ = twice sum of the series.

One half of 5×16 = sum of the terms.

Since $5 = n$, and $16 = a+l$, $S = \frac{n(a+l)}{2}$.

1. Given $a = 3$, $l = 65$, $n = 20$, to find S.

2. Given $a = 2$, $l = 50$, $n = 24$, to find S.

3. If a boy saved 1 cent on Jan. 1, and 2 cents on Jan. 2, and 3 cents on Jan. 3, and so on through the year. How much did he save during the year 1900?

4. How many strokes does a clock strike in 12 hours?

In computing Simple Interest, the principal is the *first term* of an arithmetical series, the interest for 1 year is the *common difference*, the number of years plus 1 is the *number of terms*, and the amount is the *last term*.

5. The amount of $300 at simple interest for 10 years is $450. What is the rate per cent?

FALLING BODIES.

Scientists have determined that a falling body *in a vacuum* moves by its own weight $16\frac{1}{12}$ feet during the first second, $48\frac{1}{4}$ feet in the second second, $80\frac{5}{12}$ feet in the third second, and so on. These spaces form an arithmetical progression, of which the first term is $16\frac{1}{12}$ feet, the common difference is $32\frac{1}{6}$ feet, the number of terms is the number of seconds the body falls, the last term is the distance it falls the last second, and the sum of the terms is the entire distance it falls. In the following problems *resistance by the air* is not considered.

6. A body falls 12 seconds. What distance does it fall in the last second? What is the entire distance?

7. A body falls 15 seconds. Find the distance it falls in the last second. What is the entire distance?

8. Find the distance a body falls in 10 seconds.

GEOMETRICAL PROGRESSION.

A series of numbers which succeed each other by a constant multiplier, or ratio, is called a *Geometrical Progression*.

When the ratio is greater than a unit, the series is called an *ascending* Geometrical Progression; when less than a unit, the series is called a *descending* Geometrical Progression. Thus, 1, 3, 9, 27, 81, etc., is an ascending geometrical progression, whose ratio is 3. And 1, $\frac{1}{4}$, $\frac{1}{16}$, $\frac{1}{64}$, etc., is a descending geometrical progression, whose ratio is $\frac{1}{4}$.

In geometrical, as in arithmetical progression, there are five things to be considered—

(1) *The first term.* (2) *The last term.* (3) *The common ratio.* (4) *The number of terms.* (5) *The sum of all the terms.*

These quantities are so related to each other, that any three of them being given, the remaining two may be found. For convenience in statement: a = first term, l = last term, r = ratio, n = number of terms, S = sum of all the terms.

Let 2 be the first term and 4 the ratio of a series. Then—

$$2 = \text{first term.}$$
$$2\times4 = 8 = \text{second term.}$$
$$2\times4\times4 = 2\times4^2 = 32 = \text{third term.}$$
$$2\times4\times4\times4 = 2\times4^3 = 128 = \text{fourth term.}$$

It is seen that the second term is found by multiplying the first term by the ratio; the third term, by multiplying the first by the square of the ratio; the fourth, by multiplying the first by the cube of the ratio, the *index of the power* of the ratio always being one less than the number of the term sought. A similar explanation may be given when the series is descending. Hence, to find the last term of a geometrical series—

Multiply the first term by that power of the ratio whose index is equal to the number of terms less one.

1. Given $a = 3$, $r = 2$, $n = 4$, to find l.

2. Given $a = 64$, $r = \frac{1}{2}$, $n = 6$, to find l.

3. Given $a = 5$, $r = 2$, $n = 8$, to find l.

4. Given $a = 4$, $r = 2$, $n = 10$, to find l.

Other formulas may be derived as follows—

(1) $l = ar^{n-1}$.

Divide by a = (2) $\frac{l}{a} = r^{n-1}$.

Divide (1) by r^{n-1} = (3) $\frac{l}{r^{n-1}} = a$.

1. Given $a = 5$, $l = 625$, $n = 4$, to find r.

2. Given $l = 324$, $r = 3$, $n = 5$, to find a.

3. The first term is 2, the ratio is 3, and number of terms 5. Required the sum of the terms.

SOLUTION.

(1) Sum = $2+6+18+54+162$,

Multiply (1) by ratio = (2) = $6+18+54+162+486$,

Subtract (1) from 2 = (3) = $486-2$,

(4) Sum = $\frac{486-2}{2} = 242$.

Hence, to find the sum of the terms, *multiply the last term by the ratio, subtract the first term, and divide the remainder by the ratio, less one.*

Or, expressed in algebraic formula, $S = \frac{lr-a}{r-1}$.

4. Given $a = 2$, $l = 2000$, $r = 10$, to find S.

5. What is the sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.?

NOTE.—When a series is decreasing, and the number of terms is infinite, the last term is 0.

6. What is the sum of the infinite series, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, etc.?

7. What debt will be discharged by 12 monthly payments, the first payment being \$2, the second \$4, the third \$8, etc.

8. A father gave his son 1 cent at birth, doubling it on each succeeding birthday. How much was the son worth when he became 21 years of age?

9. A person in business found that he was able to double his capital once in three years. If he commenced business with \$1,000, what did his capital amount to at the end of the twelfth year?

COMPOUND INTEREST.

COMPOUND INTEREST is the interest on both the principal and its unpaid interest added to it at stated intervals.

Interest may become due, and made a part of the principal according to agreement, at the end of each year, half-year, quarter, or other period of time.

1. What is the compound interest and the amount of $500 for 2 years 7 months 12 days, at 6 per cent?

SOLUTION.

Principal for first year	$500.00
Interest for first year	30.00
Principal for second year	$530.00
Interest for second year	31.80
Principal for 6 months 12 days	$561.80
Interest for 6 months 12 days	17.98
Compound amount for 2 years 6 months 12 days	$579.78
Given principal	500.00
Compound interest for 2 years 6 months 12 days	$ 79.78

2. What is the amount at compound interest of $400 for 2 years 8 months, at 4 per cent per annum?

3. What is the compound interest of $600 for 3 years, at 7 per cent?

4. What is the amount of $5,000 at compound interest for 4 years 10 months 12 days, at 6 per cent?

5. What is the amount of $4,000 at compound interest for 2 years 6 months 18 days, at 6 per cent?

6. What is the compound interest of $10,000 for 2 years 6 months and 18 days, at 6 per cent?

7. What is the amount of $100 at 6% per annum, compound interest, for two years, the interest being payable semi-annually?

8. What is the compound interest of $630 for 4 years, at 5%?

9. What is the amount, at compound interest, of $500, for 3 years 11 months, at 5%?

ANNUAL INTEREST.

In some States when notes are written "with interest annually," the interest, if it is not paid when it is due, draws simple interest until paid.

1.

$350. OAKLAND, CAL., MAY 17, 1903.

On demand, I promise to pay to J. J. Wood, or order, three hundred fifty dollars, with interest annually, value received.

F. MCCARTY.

What is due on this note May 17, 1907?

SOLUTION.

Int. due on principal annually...$350×0.06=	$21.00	
Principal.....	=	$350.00
Total annual interest..... 4×$21=		84.00
Interest for 3 yrs. on the $21, due May 17, 1904...=	$3.78	
Interest for 2 yrs. " " " " 1905...=	2.52	
Interest for 1 yr. " " " " 1906...=	1.26	
Interest for 6 yrs. on the deferred interest.....	=	7.56
Total amount due May 17, 1907.....	=	$441.56

NOTE.—**The simple interest on the several interests due annually, can be computed by one operation, by computing the interest on the interest due annually, for a number of years equal to the sum of the number of years the several annual interest payments run.**

What is the annual interest and amount of—

2. $2,000, for 4 years 6 months, at 8 per cent?

3. $1,530, for 6 years, at 7 per cent?

4. $1,320, for 3 years 9 months, at 6 per cent?

5. $ 930, for 6 years 3 months, at 5 per cent?

6. What is the difference between the annual and the compound interest of $1,600, for 5 years, at 6 per cent?

7. Find the amount of $850, for 4 years, at 4%, interest payable annually?

8. What amount is due July 4, 1897, on a note for $1,640, dated January 2, 1895, at 5% interest annually?

TWO UNKNOWN QUANTITIES.

It is often more convenient, and sometimes necessary, to use two or more letters to represent two or more unknown quantities.

In problems of two unknown quantities, it is seen that there are given *two* conditions, which give rise to *two* equations, each independent of the other. *Independent equations* are such as cannot be derived from one another.

To find the value of several unknown quantities, there must be as many independent equations in which the unknown quantities occur as there are unknown quantities.

In the solution of such problems, by properly combining equations one of the unknown quantities disappears. *The quantity is said to be eliminated, and the process is called elimination.* There are three methods of elimination—by comparison, by substitution, by addition and subtraction.

ELIMINATION BY COMPARISON.

Elimination by comparison is finding the value of the same unknown quantity in two different equations, in terms of the other quantity, and then placing these values equal to each other.

1. $4x+ y = 51,$

$x+4y = 24,$

Find the value of x and y.

SOLUTION—Elimination by comparison.

	(1)	$4x+ y$	$= 51.$
	(2)	$x+4y$	$= 24.$
Transposing (1) =	(3)	$4x$	$= 51-y.$
	(4)	x	$= \frac{51-y}{4}.$
Transposing (2) =	(5)	x	$= 24-4y.$
Axiom 7,	(6)	$\frac{51-y}{4}$	$= 24-4y.$
	(7)	$51-y$	$= 96-16y.$
	(8)	$15y$	$= 45.$
	(9)	y	$= 3.$
Subtracting in (2) =	(10)	x	$= 24-12 = 12.$

ELIMINATION BY SUBSTITUTION.

Elimination by substitution is finding the value of one of the unknown quantities in one of the equations, in terms of the other quantity, and substituting this instead of the quantity, in the other equation.

1. $4x+y=51.$
$x+4y=24.$

Find the value of x and y.

SOLUTION—Elimination by substitution.

(1) $4x+y=51,$
(2) $x+4y=24,$
Transposing (1) = (3) $4x=51-y,$
(4) $x=\frac{51-y}{4}.$
Substituting in (2) = (5) $\frac{51-y}{4}+4y=24,$
(6) $51-y+16y=96,$
(7) $15y=45,$
(8) $y=3,$ etc.

ELIMINATION BY ADDITION AND SUBTRACTION.

Elimination by addition and subtraction is multiplying or dividing two equations, so as to render the coefficient of one of the unknown quantities, the same in both; and then, by adding or subtracting, to cause the term containing it to disappear.

2. $4x+y=51,$
$x+4y=24$

Find the value of x and y.

SOLUTION—By addition or subtraction.

(1) $4x+y=51,$
(2) $x+4y=24,$
Multiply (1) by 4, (3) $16x+4y=204,$
(3) minus (2) = (4) $15x=180,$
(5) $x=12,$ etc.

Solve the following problems by using one unknown quantity, and also by using two unknown quantities—

1. The sum of the ages of two brothers is 22 years, and the difference of twice their ages is 12 years. How old are they?

SOLUTION I.—One unknown quantity.

Let (1) x = age of one,
and (2) $22-x$ = age of the other,
(3) $22-2x$ = the difference,
(4) $44-4x$ = 12 years,
(5) $32 = 4x$,
(6) 8 years = x, the age of one,
(7) $22-8$ = 14 years, the age of the other.

SOLUTION II.—Two unknown quantities.

Let (1) x = age of one,
and (2) y = age of the other,
(3) $x+y = 22$,
(4) $2x-2y = 12$,
(3)×2 = (5) $2x+2y = 44$,
(4)+(5) (6) $4x = 56$,
(7) $x = 14$, the age of one,
(8) $14+y = 22$,
(9) $y = 8$, the other's age.

2. The sum of two numbers divided by 3 equals 4, and their difference divided by 2 equals 4. Find the numbers.

3. Divide the number 120 into two such parts, that the one may be contained in the other $1\frac{1}{2}$ times.

4. Divide the number 91 into two such parts, that the greater divided by their difference will give 7 for a quotient.

5. Divide the number 34 into two such parts, that if 18 be subtracted from the greater, and the less be subtracted from 18. the first remainder shall be to the second as 2 to 3.

6. Divide 140 into two such parts that if 65 be subtracted from one, and 25 be added to the other, they will be in the ratio of 1 to 3.

7. State an original problem involving two unknown quantites.

Find the value of x and y in the following equations:

1. $x+y=17$
$x-y=3.$

2. $x-y=11$
$2x-3y=17.$

3. $3x-2y=7$
$2x-3y=3.$

4. $\frac{x}{3}-\frac{x}{4}=3$
$\frac{x}{2}+\frac{y}{3}=15.$

5. $3x+\frac{y}{3}=35$
$\frac{x}{2}+y=20.$

6. $2x+3y=13$
$\frac{x}{2}+10y=31.$

7. The sum of two numbers is 57, and the difference is 7. Find the numbers.

8. Find two numbers, such that $\frac{1}{2}$ of the first and $\frac{1}{3}$ of the second make 22, and $\frac{1}{4}$ of the first, and $\frac{1}{5}$ of the second make 12.

9. If the greater of two numbers be added to $\frac{1}{3}$ of the less, the sum will be 37; but if the less be diminished by $\frac{1}{4}$ of the greater, the difference will be 20. What are the numbers?

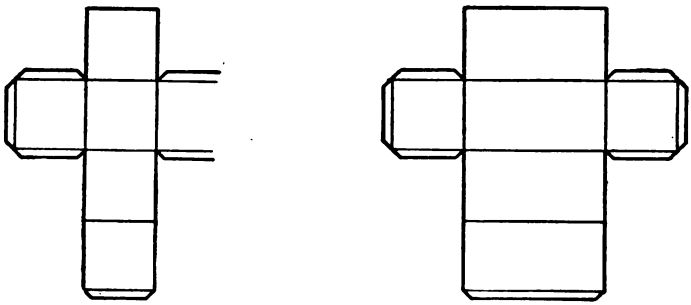
10. Find two numbers that $\frac{1}{2}$ of the first diminished by $\frac{1}{3}$ of the second are 5; and $\frac{1}{4}$ of the first diminished by $\frac{1}{5}$ of the second are 2.

11. A farmer sold two dozen eggs and 3 pounds butter for 90 cents, and 3 dozen eggs and 5 pounds butter at the same rate for $1.45. Find the price per dozen and per pound.

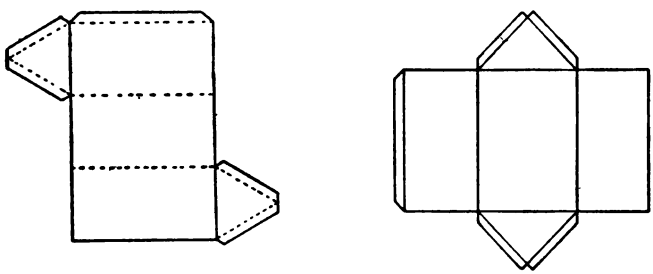
12. A and B together have $60. If B gives $5 to A they will then have the same amount. How much has each? Solve both by arithmetical analysis and by algebra.

13. If the numerator of a fraction be increased by 7 the resulting number will be 2; but if the denominator be increased by 4 the resulting fraction will be equal to one half. Find the fraction.

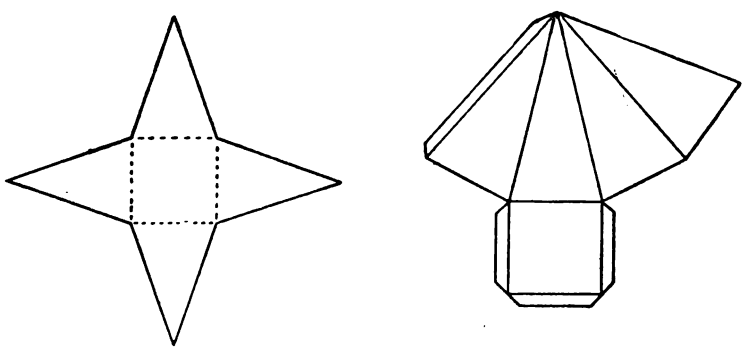
14. If 4 be subtracted from both terms of a fraction, the resulting fraction will equal $\frac{10}{14}$, and if 5 be subtracted, the resulting fraction will equal $\frac{2}{3}$. Find the fraction.



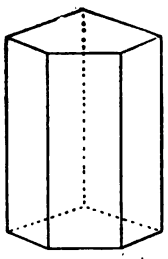
The Surface of Rectangular Solids.



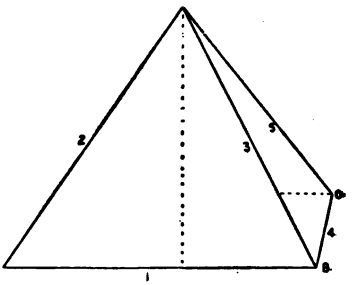
The Surface of Triangular Prisms.



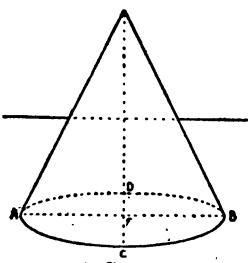
The Surface of Square Pyramids.



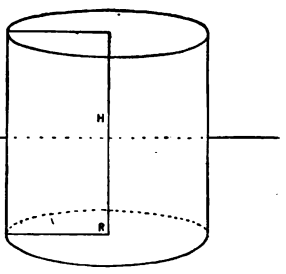
Right Prism.



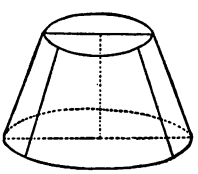
Square Pyramid.



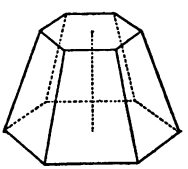
A Cone.



A Cylinder.



Frustum of a Cone.



Frustum of a Pyramid.

What dimensions are required--

1. To find the area of one face of a right prism?
2. To find the area of one face of a pyramid?
3. To find the area of the convex surface of a cone.

GEOMETRICAL SOLIDS.

Review the tables for Volume and Capacity.
Review pages 168-171; also pages 213-215, Book One.

PRINCIPLES.

1. The solid contents of a pyramid is one third that of a prism, and the solid contents of a cone one third that of a cylinder, having the same base and altitude.

2. The solid contents of a sphere is two thirds that of a cylinder whose diameter and altitude are each equal to the diameter of the sphere.

3. The capacities of similar solids are to each other as are the cubes of any one of their similar dimensions.

4. The area of similar surfaces are to each other as are the squares of their similar dimensions.

To Find the Convex Surface of a Pyramid, or Cone.—*Multiply the perimeter of the base by one half the slant height.*

To Find the Contents of a Pyramid, or Cone.—*Multiply the area of the base by one third the altitude.*

To Find the Convex Surface of a Frustum.—*Multiply the sum of the perimeters of the two bases by one half the slant height.*

To Find the Surface of a Sphere.—*Multiply the square of the diameter by 3.1416, or multiply the circumference by the diameter, or take four times the area of a circle having an equal diameter.*

To Find the Volume of a Sphere.—*Multiply the cube of the diameter by .5236.*

To Find the Volume of a Frustum.—*Multiply the sum of the areas of the two bases and the square root of their product by one third the height.*

INDEX.